# Combining perturbative and nonperturbative transverse momentum in SIDIS

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Based on:

- J.O. Gonzalez, TCR, N. Sato, Phys.Rev.D 106 (2022) 3, 034002
- F. Aslan, L. Gamberg, T. Rainaldi, TCR, *in preparation*
- J.O. Gonzalez, T. Rainaldi, TCR, *in preparation*

Workshop on MCEGs for the EIC, Friday November 18, 2022

## Balance two (somewhat) competing goals

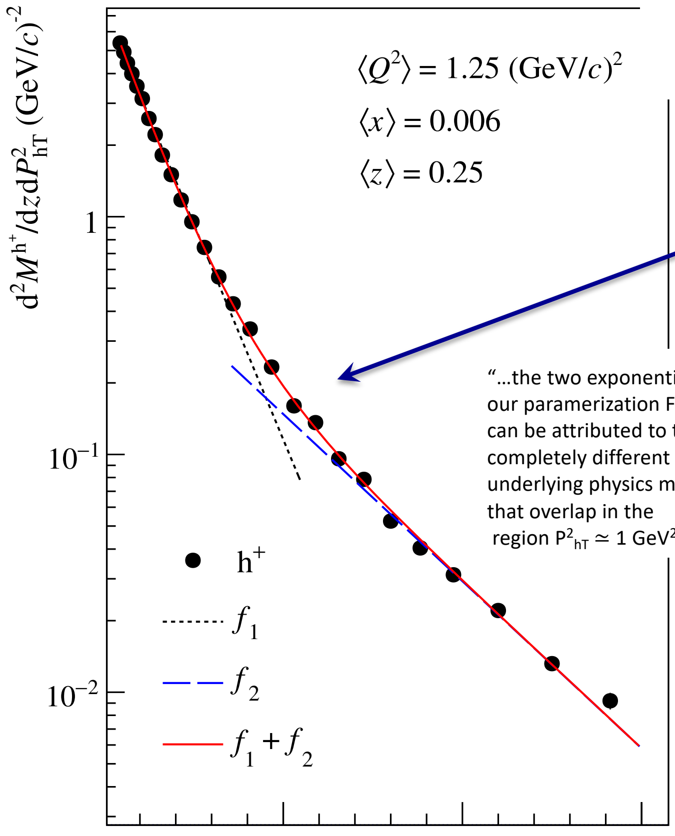
– Maximize predictive power from leading twist *collinear* pQCD as $Q \to \infty$

– Find sensitivity to hadron structure (*typical experimental scales:* $Q \approx 1 - 4$ GeVs.)

# Boundaries of regions

PHYSICAL REVIEW D **97,** 032006 (2018)

**Transverse-momentum-dependent multiplicities of charged hadrons in muon-deuteron deep inelastic scattering**



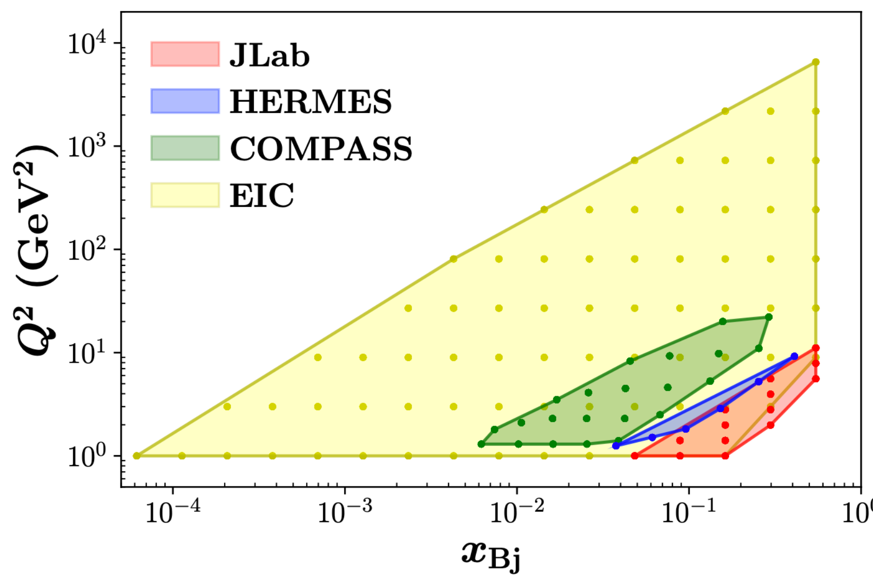

*How to treat*
*$\Lambda_{QCD} \ll q_T \ll Q$ ?*

"…the two exponential functions in our paramerization $F_1$ can be attributed to two completely different underlying physics mechanisms that overlap in the region $P^2_{hT} \simeq 1\ \mathrm{GeV}^2$. "

*Intrinsic transverse momentum*

*Radiative "tail"*

## *Q matters too*




*Boglione et al, JHEP 04 (2022) 084*

## Separating large and small transverse momentum in factorization theorems

- Expansion at small $q_T$:

$$\Gamma = \underbrace{\mathrm{App}_{\#1}\Gamma}_{\textit{TMD factorization}} + O\left(\frac{q_T^2}{Q^2}\right)$$

- Expansion at large $q_T$:

$$\Gamma = \underbrace{\mathrm{App}_{\#2}\Gamma}_{\textit{Fixed order collinear factorization}} + O\left(\frac{m^2}{q_T^2}\right)$$

## Separating large and small transverse momentum in factorization theorems

- Expansion at small $q_T$: $$\Gamma = \underbrace{\text{App}_{\#1}\Gamma}_{\textit{TMD factorization}} + O\left(\frac{q_T^2}{Q^2}\right)$$

- Expansion at large $q_T$: $$\Gamma = \underbrace{\text{App}_{\#2}\Gamma}_{\textit{Fixed order collinear factorization}} + O\left(\frac{m^2}{q_T^2}\right)$$

- Explicit error: $$\begin{aligned}\Gamma &= \text{App}_{\#1}\Gamma + \left[\Gamma - \text{App}_{\#1}\Gamma\right] \\ &= \text{App}_{\#1}\Gamma + \text{App}_{\#2}\left[\Gamma - \text{App}_{\#1}\Gamma\right] + O\left(\frac{q_T^2}{Q^2} \times \frac{m^2}{q_T^2}\right) \\ &= \text{App}_{\#1}\Gamma + \text{App}_{\#2}\Gamma - \text{App}_{\#2}\text{App}_{\#1}\Gamma + O\left(\frac{m^2}{Q^2}\right)\end{aligned}$$

# Collinear factorization

- TMD version

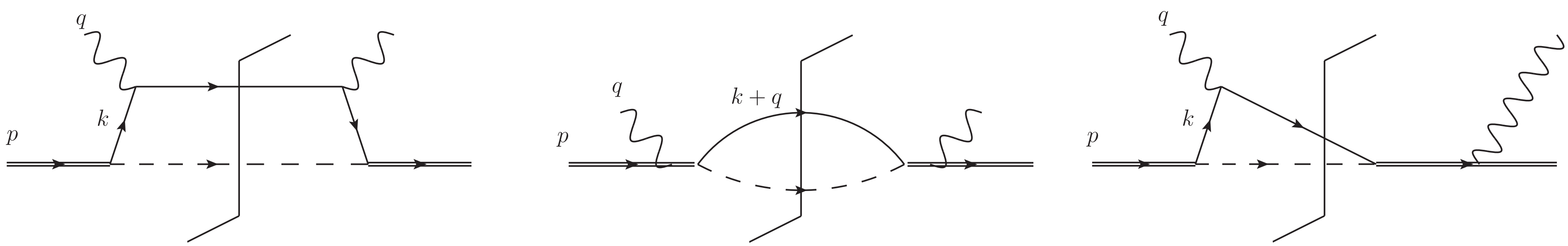
$$\Gamma = \underbrace{\mathrm{App}_{\#1}\Gamma}_{\text{TMD factorization}} + \underbrace{\mathrm{App}_{\#2}\Gamma}_{\text{Fixed order collinear factorization}} - \underbrace{\mathrm{App}_{\#2}\mathrm{App}_{\#1}\Gamma}_{\text{Asymptotic term}} + O\left(\frac{m^2}{Q^2}\right)$$

- Inclusive DIS

$$\sum \int \mathrm{d}^2\mathbf{q}_T \, \Gamma = \hat{\Gamma} \otimes f + \left(\frac{m^2}{Q^2}\right)$$

## How it works in an easy case

- Stress-test DIS factorization in other finite-range, renormalizable theories

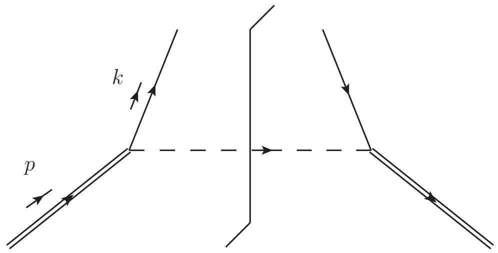



(a) (b) (c)

$$\mathcal{L}_{\rm int} = -\lambda\, \overline{\Psi}_N\, \psi_q\, \phi \; + \; \text{H.C.}$$

- Exact $O(\lambda^2)$ (SI)DIS cross section is easy to calculate

## Parton densities with MS-bar renormalization




- Collinear

$$f_{0,i/p}(\xi) = \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-}\ \langle p|\, \bar{\psi}_{0,i}(0, w^-, \mathbf{0}_\mathrm{T}) \frac{\gamma^+}{2} \psi_{0,i}(0,0,\mathbf{0}_\mathrm{T})\, |p\rangle$$

$$f_{i/p}(\xi;\mu) = Z_{i/i'} \otimes f_{0,i'/p}$$

- Transverse momentum dependent (TMD)

$$f_{0,i/p}(\xi, \boldsymbol{k}_\mathrm{T}) = \int \frac{\mathrm{d}w^-\ \mathrm{d}^2\boldsymbol{w}_\mathrm{T}}{(2\pi)^3}\, e^{-i\xi p^+ w^- + i\boldsymbol{k}_\mathrm{T}\cdot\boldsymbol{w}_\mathrm{T}}\ \langle p|\, \bar{\psi}_{0,i}(0, w^-, \boldsymbol{w}_\mathrm{T}) \frac{\gamma^+}{2} \psi_{0,i}(0,0,\mathbf{0}_\mathrm{T})\, |p\rangle$$

$$f_{0,i/p}(\xi, \boldsymbol{k}_\mathrm{T}) = Z_2 f_{i/p}(\xi, \boldsymbol{k}_\mathrm{T};\mu)$$

# Parton densities with MS-bar renormalization

- Collinear

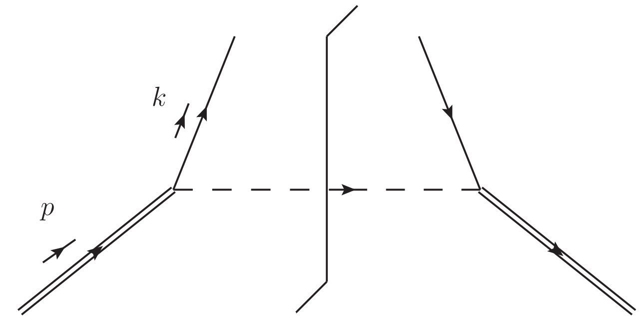
$$f_{q/p}^{(1)}(\xi;\mu) \overset{\xi \neq 1}{=} a_\lambda(\mu)(1-\xi)\left(\frac{\chi(\xi)^2}{\Delta(\xi)^2} + \ln\left[\frac{\mu^2}{\Delta(\xi)^2}\right] - 1\right)$$

- TMD

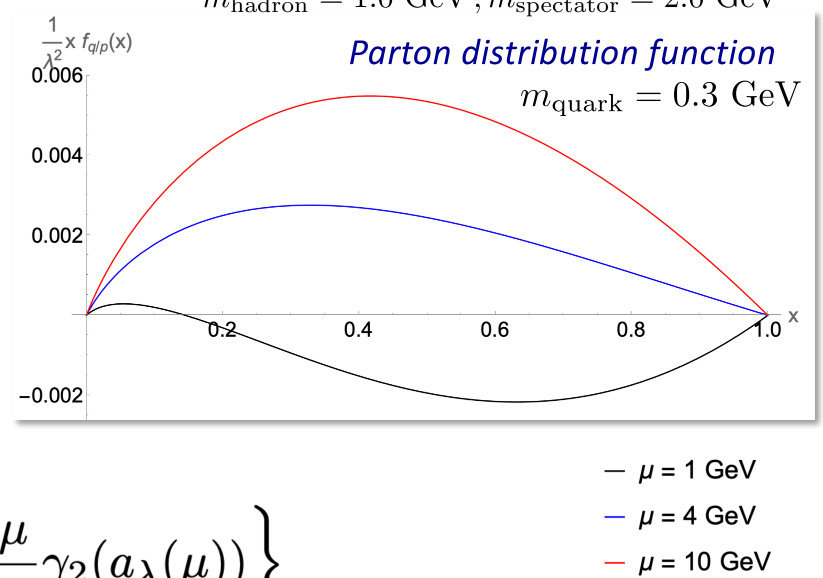
$$f_{q/p}^{(1)}(\xi, \boldsymbol{k}_\mathrm{T};\mu) = \frac{a_\lambda(\mu)}{\pi}(1-\xi)\frac{k_\mathrm{T}^2 + \chi(\xi)^2}{\left[k_\mathrm{T}^2 + \Delta(\xi)^2\right]^2}$$

- TMD evolution

$$f_{q/p}(\xi, \boldsymbol{k}_\mathrm{T};\mu) = f_{q/p}(\xi, \boldsymbol{k}_\mathrm{T};\mu_0)\exp\left\{-2\int_{\mu_0}^{\mu}\frac{\mathrm{d}\mu}{\mu}\gamma_2(a_\lambda(\mu))\right\}$$

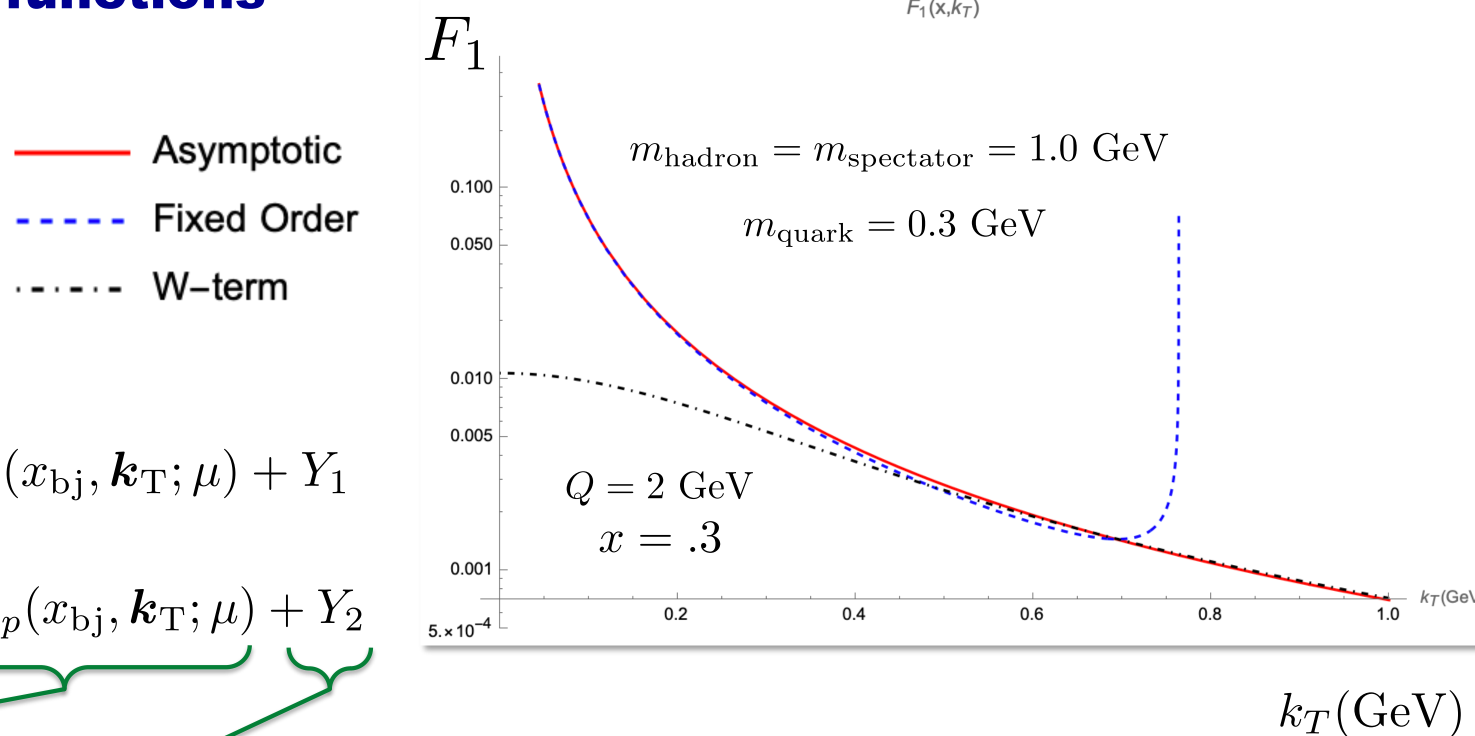



$m_\mathrm{hadron} = 1.0$ GeV , $m_\mathrm{spectator} = 2.0$ GeV



## SIDIS structure functions

- **W+Y**




$$F_1(x_{\rm bj}, Q, \boldsymbol{k}_{\rm T}) = \frac{1}{2} f_{q/p}(x_{\rm bj}, \boldsymbol{k}_{\rm T}; \mu) + Y_1$$

$$F_2(x_{\rm bj}, Q, \boldsymbol{k}_{\rm T}) = x_{\rm bj} f_{q/p}(x_{\rm bj}, \boldsymbol{k}_{\rm T}; \mu) + Y_2$$

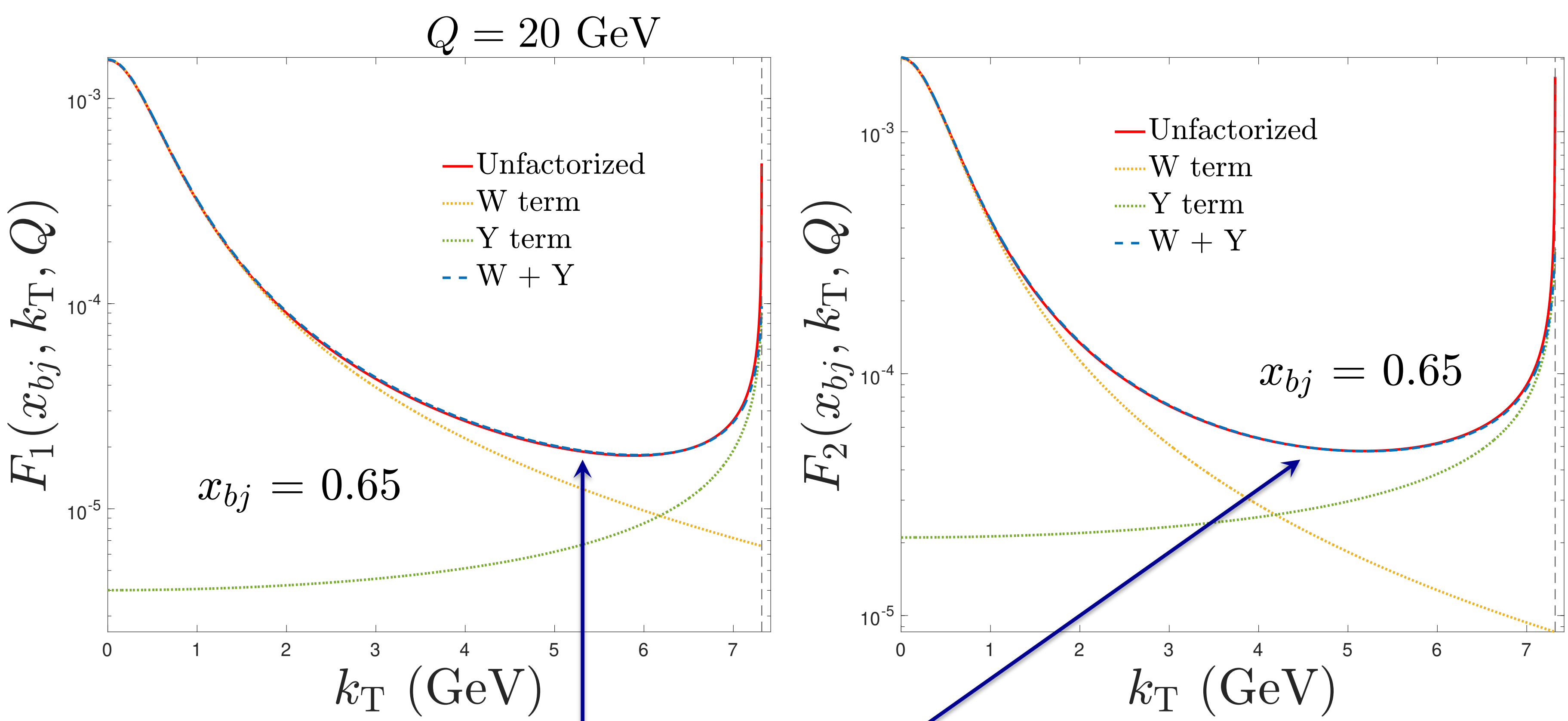
$$\Gamma = \underbrace{\mathrm{App}_{\#1}\Gamma}_{\text{TMD factorization}} + \underbrace{\mathrm{App}_{\#2}\Gamma}_{\text{Fixed order collinear factorization}} - \underbrace{\mathrm{App}_{\#2}\mathrm{App}_{\#1}\Gamma}_{\text{Asymptotic term}} + O\left(\frac{m^2}{Q^2}\right)$$

*TMD factorization*

*Fixed order collinear factorization*

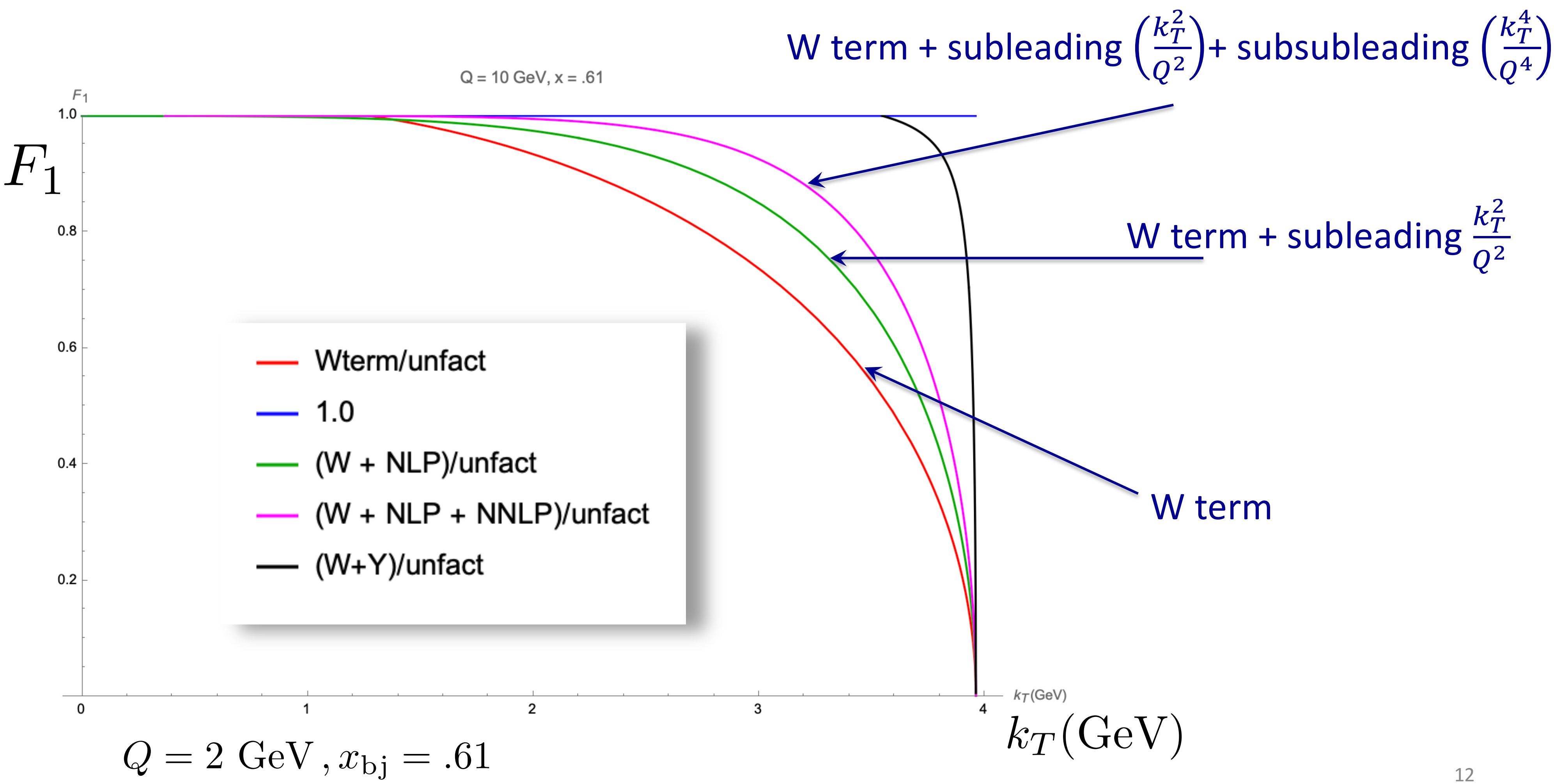
*Asymptotic term*

# SIDIS structure functions




*Important large-$k_T$ contributions*

## W+Y method vs power corrections



## Factorized collinear structure functions

$$F_1(x_{\rm bj}, Q) = \sum_i \int_{x_{\rm bj}}^1 \frac{{\rm d}\xi}{\xi} \times$$

$$\times \underbrace{\frac{1}{2}\left\{\delta\left(1-\frac{x_{\rm bj}}{\xi}\right)\delta_{qi} + a_\lambda(\mu)\left(1-\frac{x_{\rm bj}}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x_{\rm bj}}{\xi}\right)^2 - 3\frac{x_{\rm bj}}{\xi} + \frac{3}{2}}{\left(1-\frac{x_{\rm bj}}{\xi}\right)^2} - \ln\frac{4x_{\rm bj}\mu^2}{Q^2(\xi - x_{\rm bj})}\right]\delta_{pi}\right\}}_{\hat{F}_{1,q/i}(x_{\rm bj}/\xi,\mu/Q;a_\lambda(\mu))} \times$$

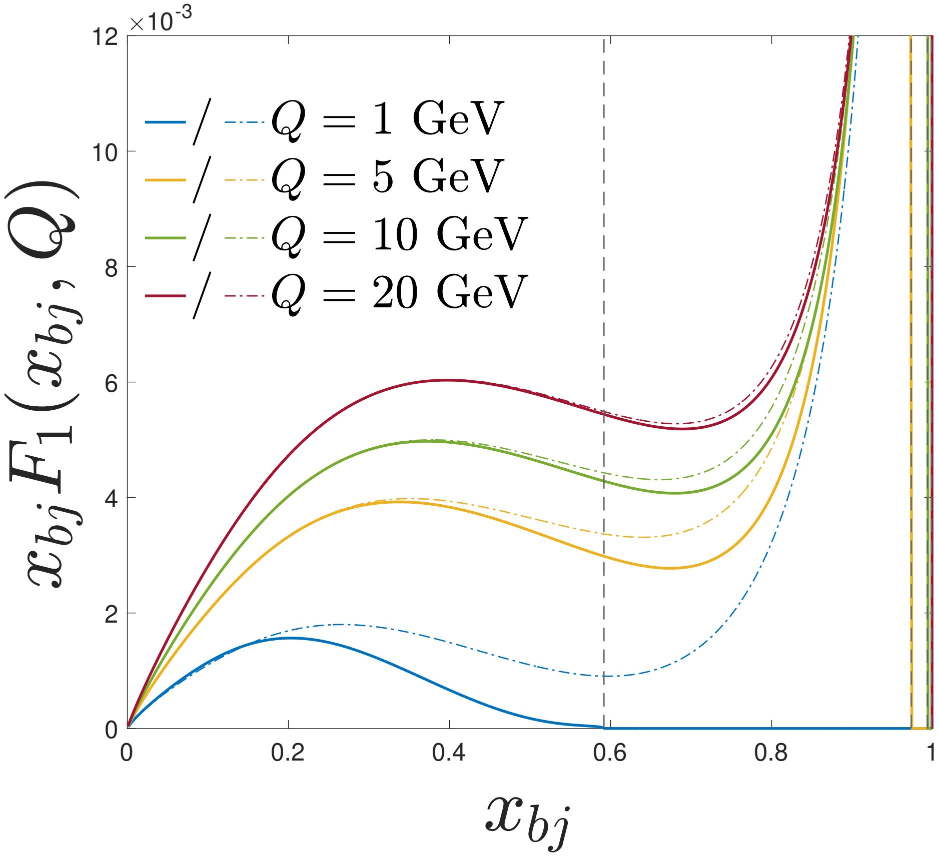
← *HARD structure function*

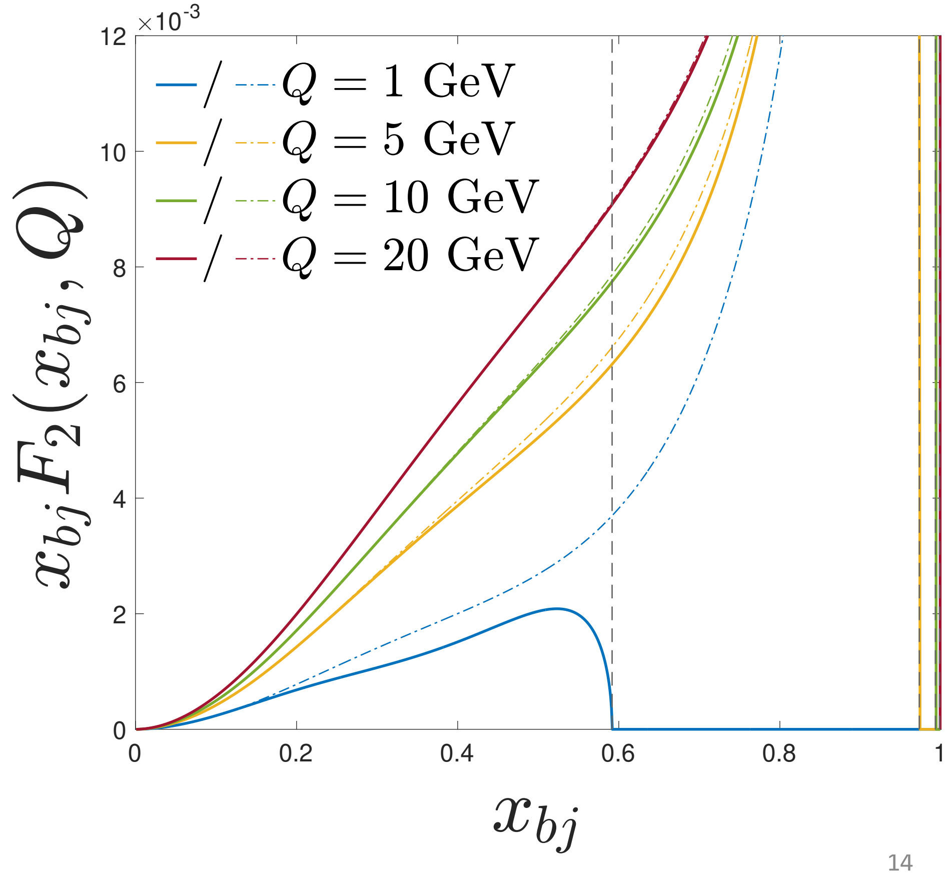
$$\times \underbrace{\left\{\delta(1-\xi)\,\delta_{ip} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q+\xi m_p)^2}{\Delta(\xi)^2} + \ln\left(\frac{\mu^2}{\Delta(\xi)^2}\right) - 1\right]\delta_{iq}\right\}}_{f_{i/p}(\xi;\mu)}$$

← *Parton distribution function*

## Unpolarized structure functions

$$m_{\text{hadron}} = m_{\text{spectator}} = 1.0 \text{ GeV}$$
$$m_{\text{quark}} = 0.3 \text{ GeV}$$



## The generalized parton model

- Only use TMD pdfs and ffs:

$$F_{1,2}(x_{\mathrm{bj}}, Q, z_h, \boldsymbol{P}_{B\mathrm{T}}) = \sum_{ij} \widehat{F}^{ij}_{1,2} \int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}}\ \mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}}\ f_{i/p}(x_{\mathrm{bj}}, \boldsymbol{k}_{1\mathrm{T}}) D_{B/j}(z_h, z_h\boldsymbol{k}_{2\mathrm{T}}) \delta^{(2)}(\boldsymbol{q}_{\mathrm{T}} + \boldsymbol{k}_{1\mathrm{T}} - \boldsymbol{k}_{2\mathrm{T}})$$

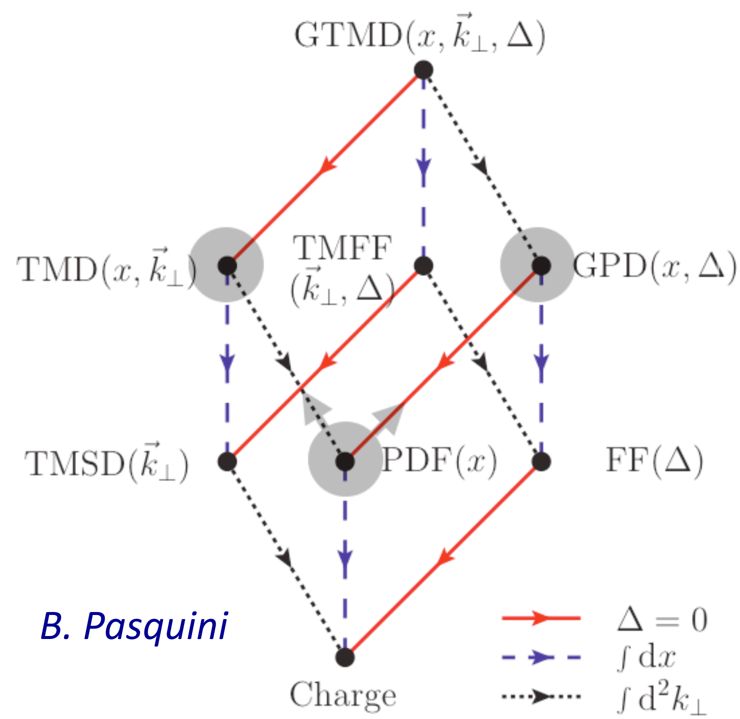
- Integrate over transverse momentum

$$\Rightarrow F_{1,2}(x_{\mathrm{bj}}, Q^2) \stackrel{??}{=} \frac{1}{4} \sum_{ij} \widehat{F}^{ij}_{1,2}\ f_{i/p}(x_{\mathrm{bj}})$$

*with*

$$\int \mathrm{d}^2\boldsymbol{k}_{1\mathrm{T}}\ f_{i/p}(x_{\mathrm{bj}}, \boldsymbol{k}_{1\mathrm{T}}) \stackrel{??}{=} f_{i/p}(x_{\mathrm{bj}})$$

$$\int \mathrm{d}z_h \int \mathrm{d}^2\boldsymbol{k}_{2\mathrm{T}}\ z_h D_{B/j}(z_h, z_h\boldsymbol{k}_{2\mathrm{T}}) \stackrel{??}{=} 1$$

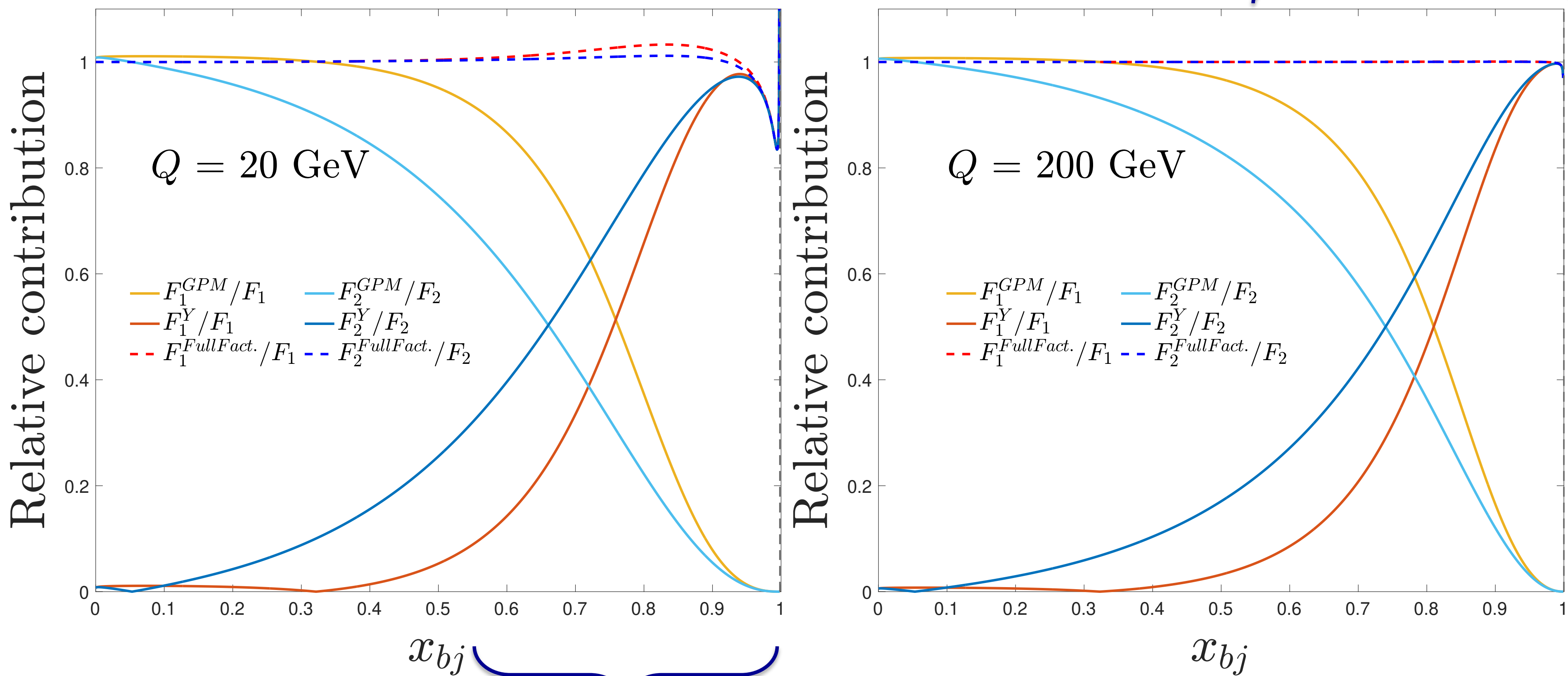



*B. Pasquini*

# The generalized parton model (GPM)

*Plots from*
*Tommaso Rainaldi*

*Large transverse momentum is important*

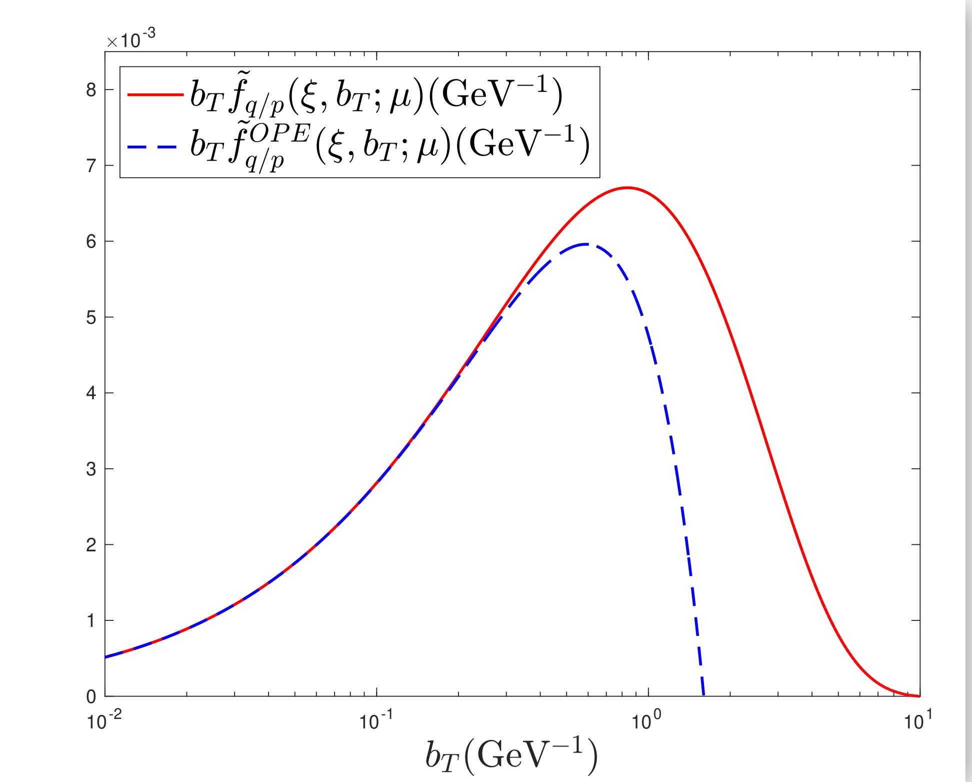



*Large transverse momentum is important*

## Cross sections in transverse coordinate space

$$\tilde{F}_1(x_{\rm bj}, Q, \boldsymbol{b}_{\rm T}) = \frac{1}{2}\tilde{f}_{q/p}(x_{\rm bj}, \boldsymbol{b}_{\rm T}; Q) + \tilde{Y}_1$$

$$\tilde{F}_2(x_{\rm bj}, Q, \boldsymbol{k}_{\rm T}) = x_{\rm bj}\tilde{f}_{q/p}(x_{\rm bj}, \boldsymbol{b}_{\rm T}; Q) + \tilde{Y}_2$$




- Operator product expansion

$$\tilde{f}_{q/p}(x_{\rm bj}, \boldsymbol{b}_{\rm T}; \mu) = \sum_j \int_{x_{\rm bj}}^1 \frac{\mathrm{d}\xi}{\xi}\tilde{\mathcal{C}}_{q/j}(x_{\rm bj}/\xi, \boldsymbol{b}_{\rm T}; \mu)\tilde{f}_{j/p}(\xi; \mu) + \mathcal{O}(m^2 b_{\rm T}^2)$$

$$= f_{q/p}(x_{\rm bj}; \mu) - a_\lambda(\mu)(1 - x_{\rm bj}) \ln\left(\frac{\mu^2 b_{\rm T}^2 e^{2\gamma_E}}{4}\right) + \cdots + \mathcal{O}(m^2 b_{\rm T}^2)$$

## Separating large and small transverse coordinates

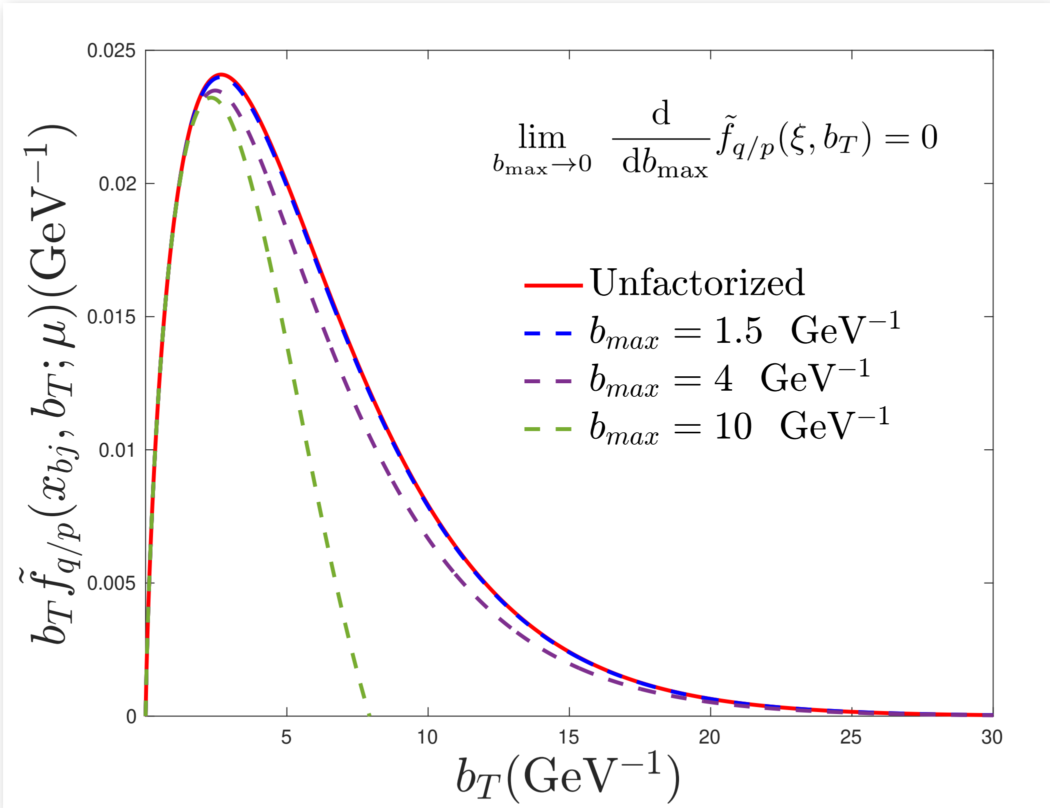
$$m_{\text{hadron}} = m_{\text{spectator}} = 1.0 \text{ GeV}$$
$$m_{\text{quark}} = 0.3 \text{ GeV}$$

- The b* method $\boldsymbol{b}_*(b_{\mathrm{T}}) = \frac{\boldsymbol{b}_{\mathrm{T}}}{\sqrt{1 + b_{\mathrm{T}}^2/b_{\max}^2}}$

$$\tilde{f}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_{\mathrm{T}}; \mu) = \tilde{f}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_*; \mu) \frac{\tilde{f}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_{\mathrm{T}}; \mu)}{\tilde{f}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_*; \mu)}$$

$$= \tilde{f}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_*; \mu) \exp\{-g_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_{\mathrm{T}})\}$$

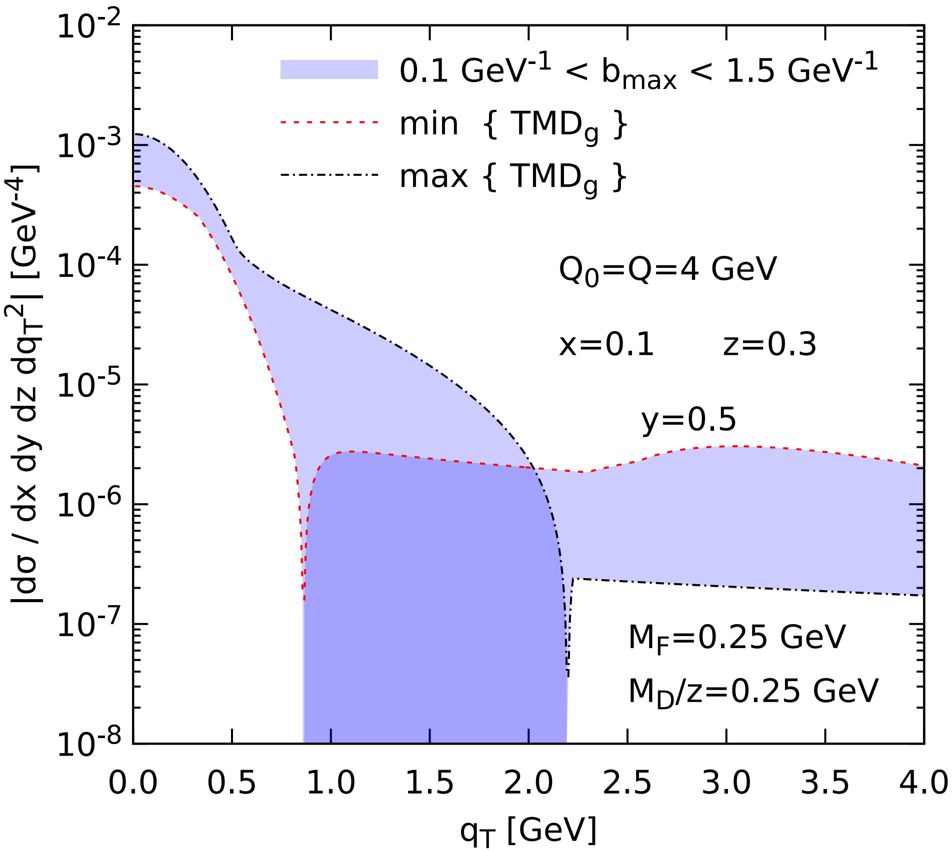
$$g_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_{\mathrm{T}}) \equiv -\ln \left( \frac{\tilde{f}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_{\mathrm{T}}; \mu)}{\tilde{f}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_*; \mu)} \right)$$



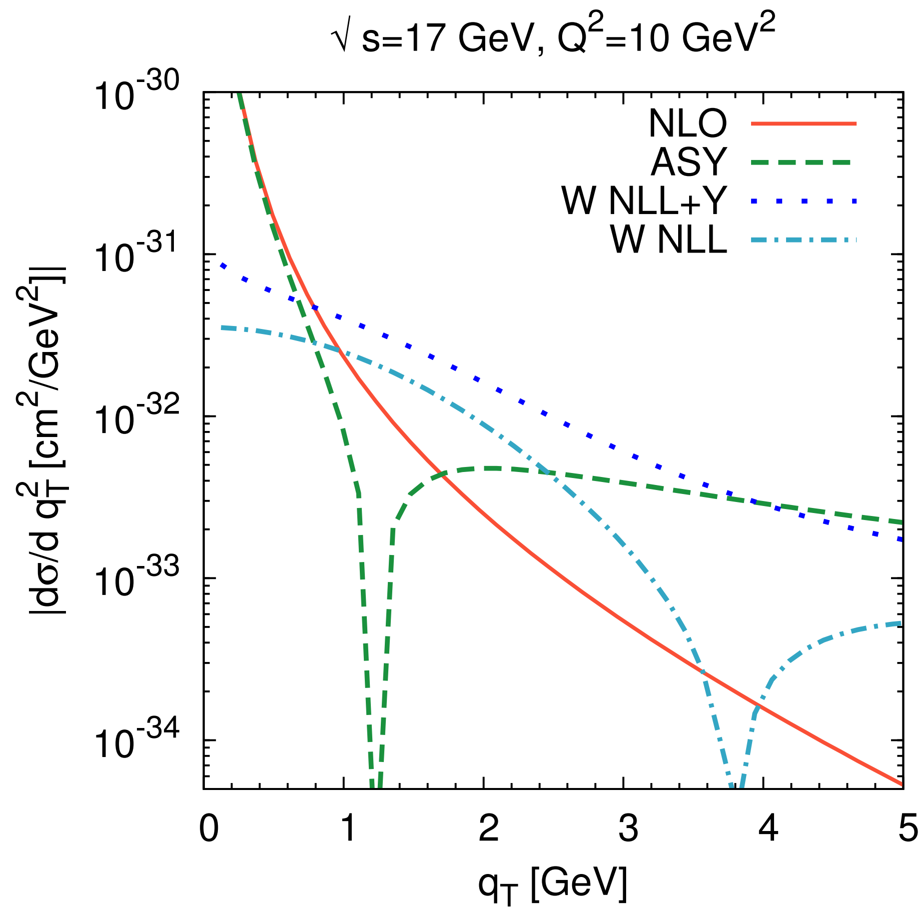

- Use the OPE

$$\tilde{f}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_{\mathrm{T}}; \mu) = \tilde{f}^{\mathrm{OPE}}_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_*; \mu) \exp\{-g_{q/p}(x_{\mathrm{bj}}, \boldsymbol{b}_{\mathrm{T}})\} + \mathcal{O}(m^2 b_{\max}^2)$$

## In QCD

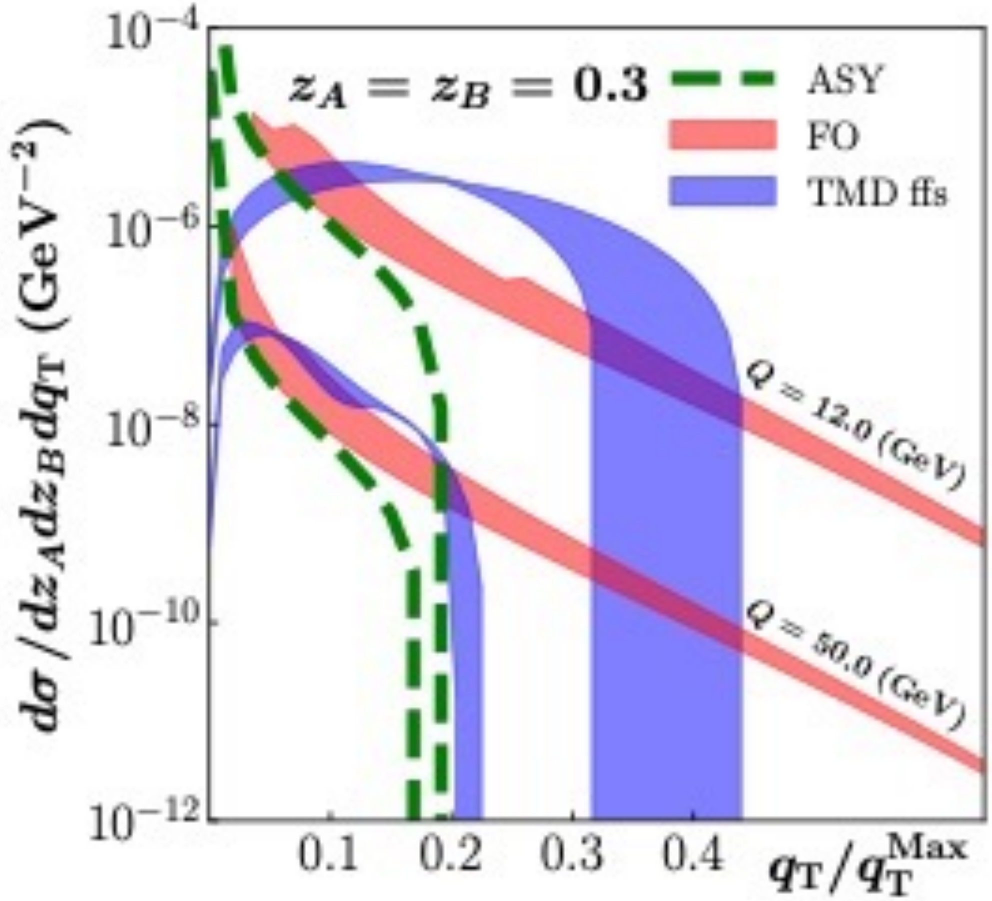



*Typical $b_{max}$ dependence in QCD*




*Boglione et al, JHEP 1502 (2015) 095*

## $e^+e^-$ annihilation




*E. Moffat, et al Phys.Rev.D 104 (2021) 5, 059904*

- Blue band:
  - from survey of non-perturbative fits
- Pink band:
  - Large transverse momentum calculation, width from varying RG scale
- Green:
  - Small $q_T/Q \to 0$ asymptote
- No overlap in the transition region for smaller Q

## Hadron structure oriented (HSO) approach

- W-term

$$W(q_T, Q_0) = \int d^2\mathbf{k}_{1T}\, d^2\mathbf{k}_{2T}\, f(x, k_{1T}; Q_0) D(z, zk_{2T}; Q_0) \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

- Evolution

$$\tilde{W}(b_T, Q) = \tilde{W}(b_T, Q_0) \underbrace{E(Q, Q_0, b_T)}_{\text{“Straightforward”}}$$

- Need input TMD pdf & ff for **all** $k_{1T}$ and $k_{2T}$

## Hadron structure oriented (HSO) approach

- Probability density/ partonic structure interpretation

$$f_{i/p}(x;Q_0) = \pi \int^{Q_0^2} \mathrm{d}k_\mathrm{T}^2 \, f_{i/p}(x, \boldsymbol{k}_\mathrm{T}; Q_0)$$

- Consistent large $k_T$ behavior (LO for now)

$$f(x, \boldsymbol{k}_\mathrm{T}; \mu_{Q_0}, Q_0^2) \stackrel{k_\mathrm{T} \approx Q_0}{\longrightarrow} \frac{1}{2\pi} \frac{1}{k_\mathrm{T}^2} \left[ A_{i/p}^f(x; \mu_{Q_0}) + B_{i/p}^f(x; \mu_{Q_0}) \ln \frac{Q_0^2}{k_\mathrm{T}^2} \right] + \frac{1}{2\pi} \frac{1}{k_\mathrm{T}^2} A_{g/p}^f(x; \mu_{Q_0})$$

## Impose a partonic interpretation at the input scale

***(Integral constraints even for g-function!)***

- HSO constrained

Gaussian NP core

$$f_{i/p}(x,\boldsymbol{k}_\mathrm{T};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_\mathrm{T}^2+m_{f_{i/p}}^2}\left[A_{i/p}^f(x;\mu_{Q_0}) + B_{i/p}^f(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_\mathrm{T}^2+m_{f_{i/p}}^2}\right] + \frac{1}{2\pi}\frac{1}{k_\mathrm{T}^2+m_{f_{g/p}}^2}A_{g/p}^f(x;\mu_{Q_0}) + \frac{C_{i/p}^f}{\pi M_{f_{i/p}}^2}e^{-k_\mathrm{T}^2/M_{f_{i/p}}^2}$$

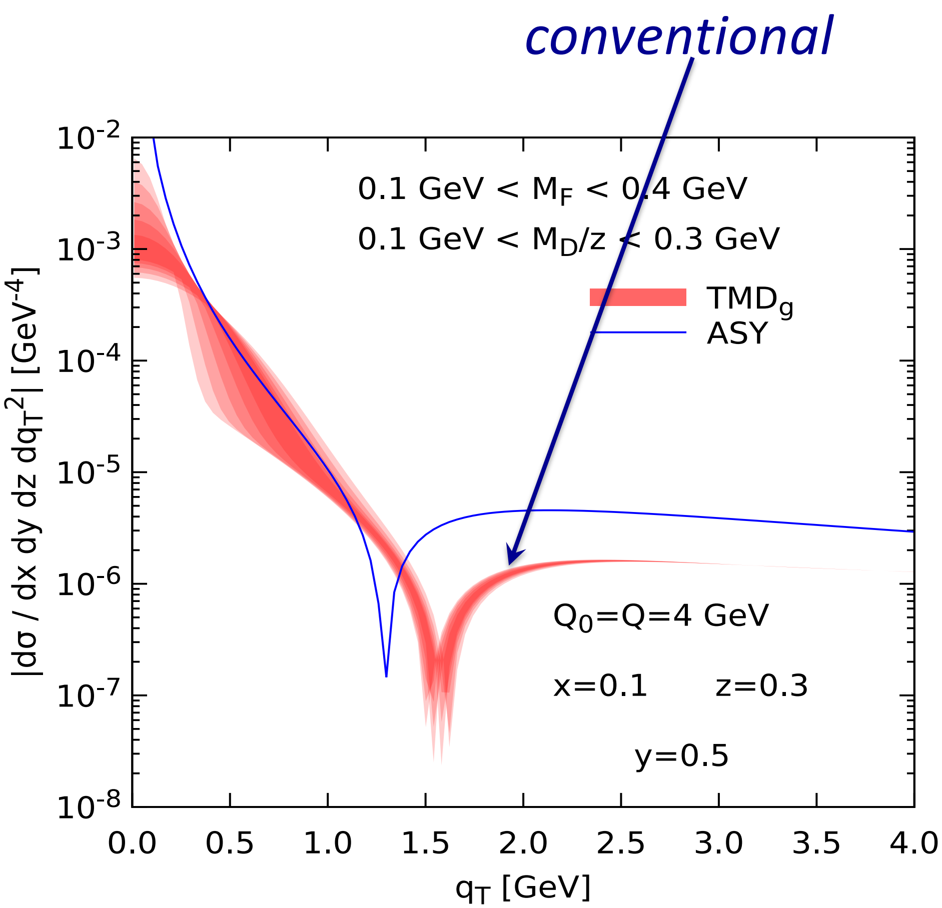
Integral constraint

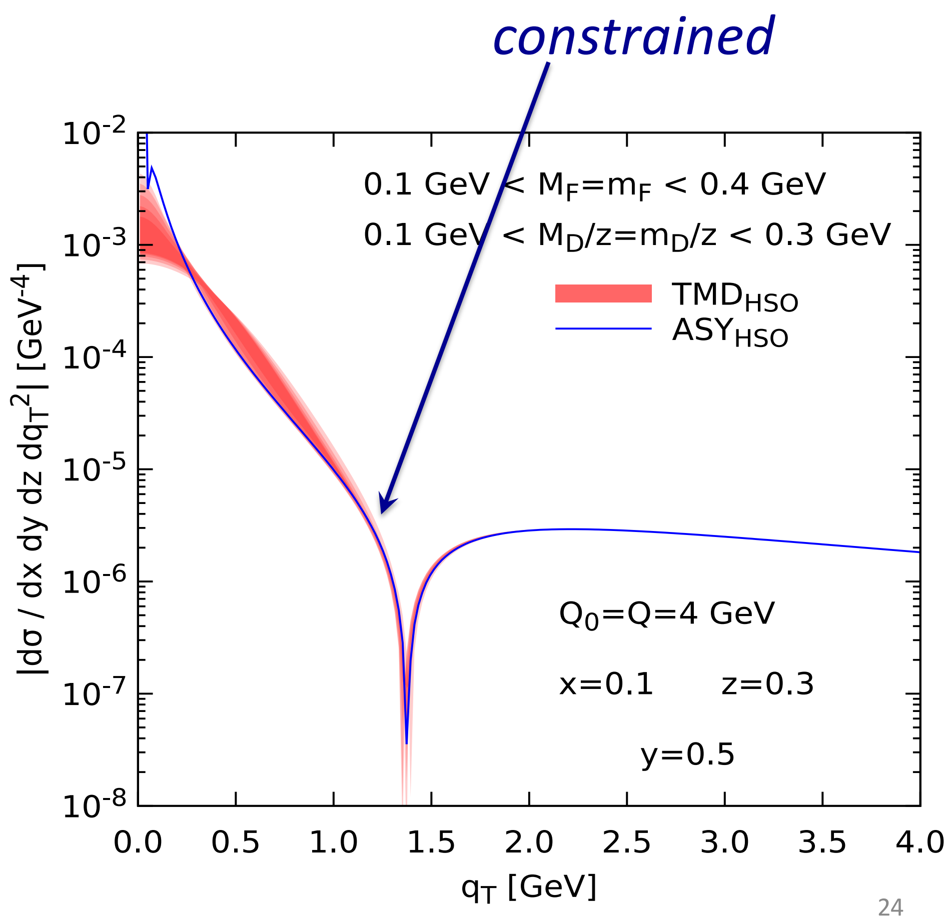
$$C_{i/p}^f \equiv f_{i/p}(x;\mu_{Q_0}) - A_{i/p}^f(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i/p}}}\right) - B_{i/p}^f(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{i/p}}}\right)\ln\left(\frac{Q_0^2}{\mu_{Q_0}m_{f_{i/p}}}\right) - A_{g/p}^f(x;\mu_{Q_0})\ln\left(\frac{\mu_{Q_0}}{m_{f_{g/p}}}\right) + \frac{\alpha_s(\mu_{Q_0})}{2\pi}\left\{[\mathcal{C}_\Delta^{i/p}\otimes f_{i/p}](x;\mu_{Q_0}) + [\mathcal{C}_\Delta^{g/p}\otimes f_{g/p}](x;\mu_{Q_0})\right\}$$

- Standard version

$$g_{j/p}(x_\mathrm{bj},b_\mathrm{T}) = \frac{1}{4}M_{g_F}^2\,b_\mathrm{T}^2\,, \qquad g_{h/j}(z_\mathrm{h},b_\mathrm{T}) = \frac{1}{4\,z_\mathrm{h}^2}M_{g_D}^2\,b_\mathrm{T}^2$$

# Impose the partonic interpretation at the input scale



## Conclusions

- Monte Carlo question: Importance of ”large” vs “small” transverse momentum?

- What does large/small transverse momentum mean?

- Answer depends on how simulation is to be used:
  - Testing notions of intrinsicness / partonic structure
  - Precision at high energies?