A hadron structure oriented approach to TMD phenomenology

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Based on

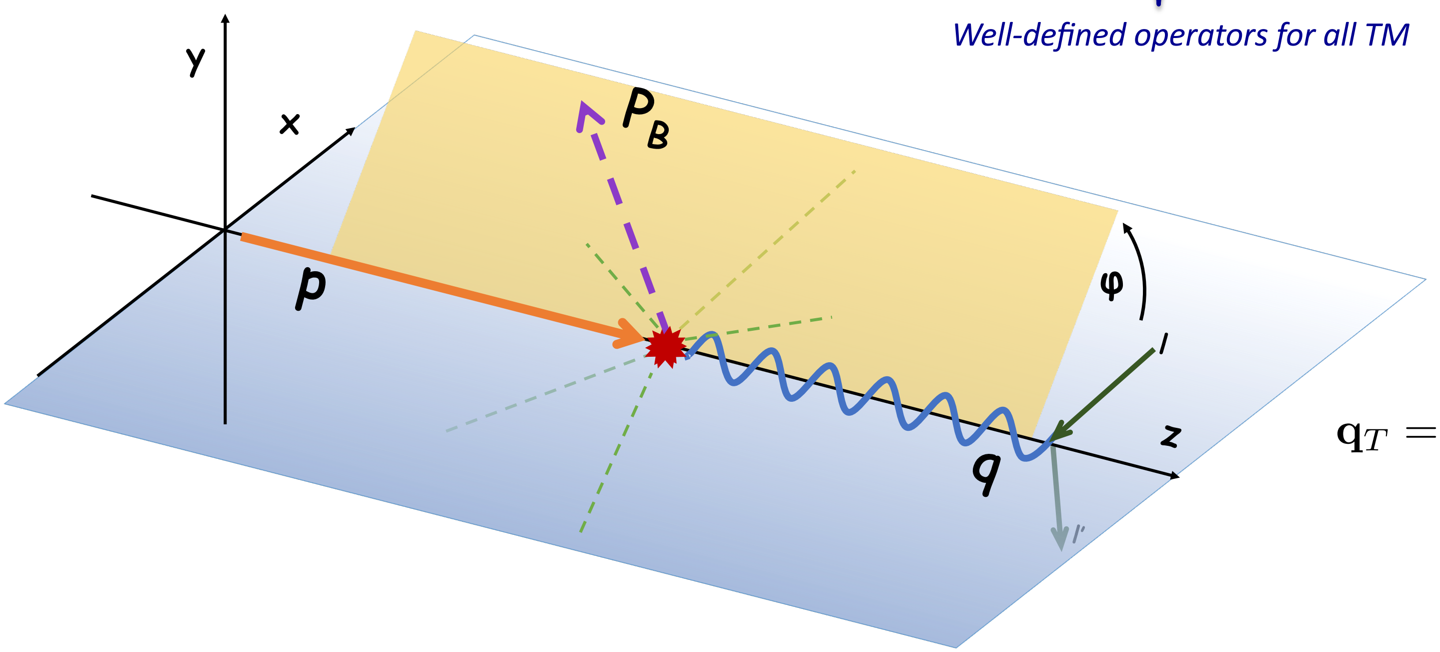
- J.O. Gonzalez, TCR, N. Sato, Phys.Rev.D 106 (2022) 3, 034002
- J.O. Gonzalez, T. Rainaldi, TCR, in preparation

Today

TMD factorization & SIDIS $\left(for\ \frac{q_T}{Q} \ll 1\right)$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T}\underbrace{f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2)D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\text{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$$

+ power suppressed



$$\mathbf{q}_T = -\frac{\mathbf{P}_{BT}}{z}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\int \frac{\mathrm{d}^2\mathbf{b}_T}{(2\pi)^2}e^{-i\mathbf{b}_T\cdot\mathbf{q}_T}\tilde{f}_{j/p}(x,\mathbf{b}_T;\mu_Q,Q^2)\tilde{D}_{h/j}(z,\mathbf{b}_T;\mu_Q,Q^2)$$

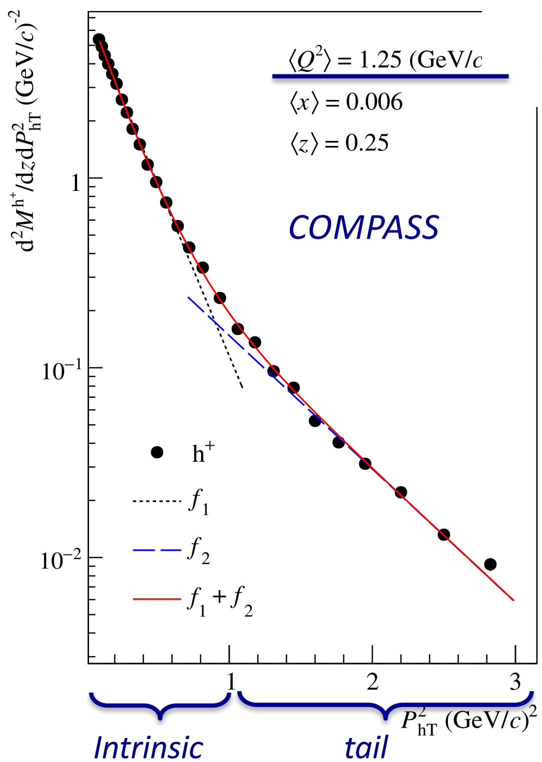
+ power suppressed

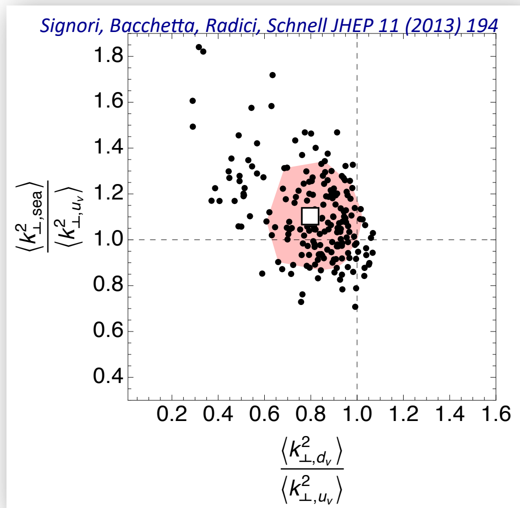
Nonperturbative structures in pheno

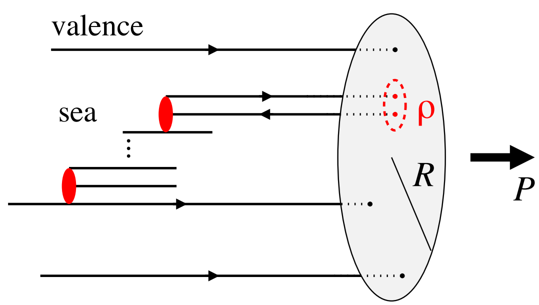
"While significant effort has been put into the study of W(b) ***at large b*** *[36, 42, 43, 44], none … adequately describe the observed Z boson distribution without introducing free parameters."*

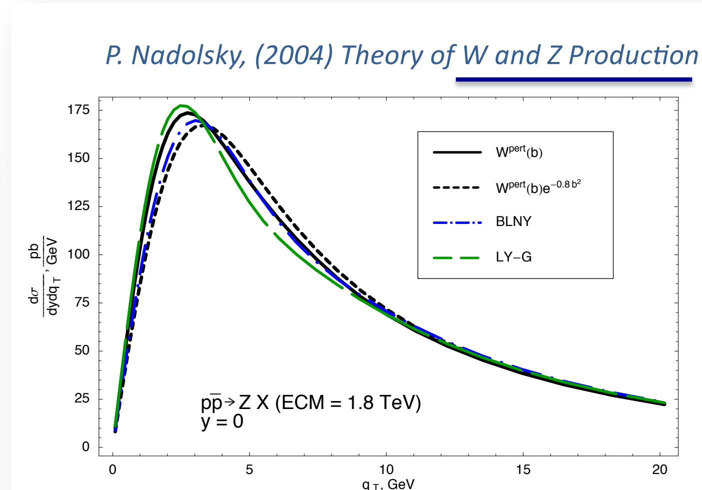
PHYSICAL REVIEW D **97,** 032006 (2018)

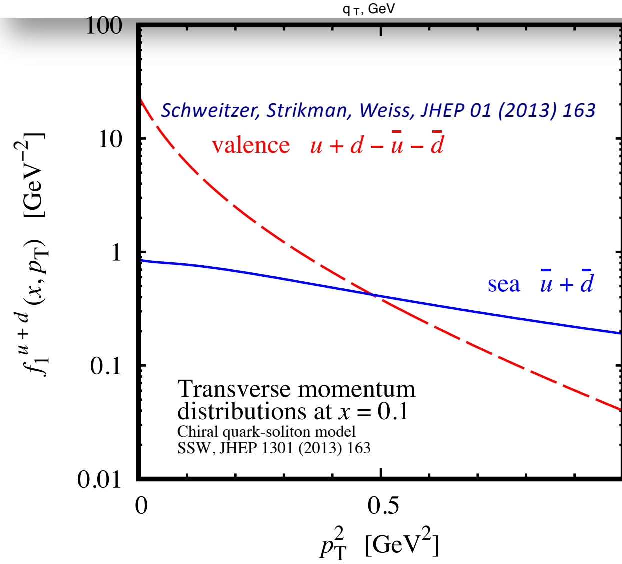
Transverse-momentum-dependent multiplicities of charged hadrons in muon-deuteron deep inelastic scattering











"… the two exponential functions in our parametrizations F_1 can be attributed to two completely different underlying physics mechanisms that overlap in the region $P_{hT} \simeq 1$ GeV^2."

Conventional organization

$$\frac{\partial \ln \tilde{f}_{j/p}(x,b_T;\mu,\zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T;\mu)$$

$$\frac{\mathrm{d}\ln \tilde{f}_{j/p}(x,b_T;\mu,\zeta)}{\mathrm{d}\ln \mu} = \gamma(\alpha_s(\mu);\zeta/\mu^2)$$

$$\frac{\mathrm{d}\tilde{K}(b_T;\mu)}{\mathrm{d}\ln \mu} = -\gamma_K(\alpha_s(\mu))$$

$$\tilde{f}_{j/p}(x,b_T;\mu,\zeta) = \int_x^1 \frac{\mathrm{d}\xi}{\xi}\tilde{C}_{j/k}(x/\xi,b_T;\zeta,\mu,\alpha_s(\mu)) f_{k/p}(\xi;\mu) + O\left(b_T\Lambda_{\mathrm{QCD}}\right)$$

Or

$$f_{j/p}(x,k_T;\mu,\zeta) = \frac{1}{k_T^2}\left[\int_x^1 \frac{\mathrm{d}\xi}{\xi}C_{j/k}(x/\xi,k_T;\zeta,\mu,\alpha_s(\mu)) f_{k/p}(\xi;\mu) + O\left(\frac{\Lambda_{\mathrm{QCD}}}{k_T}\right)\right]$$

$$\tilde{K}(b_T;\mu) = \tilde{K}_{\mathrm{pert}}\left(b_T\mu,\alpha_s(\mu)\right) + O\left(b_T\Lambda_{\mathrm{QCD}}\right)$$

Steps:

1) Solve evolution equations to relate overall SIDIS hard scale ($\mu_Q = Q$) to input scale ($\mu_{Q_0} = Q_0$)
2) How to use small $b_T \ll 1/\Lambda_{QCD}$ collinear factorization?
   - Partition small ($b_T < b_{\max}$) & large ($b_T > b_{\max}$) regions with a b_*
   - Define hard scale $\mu_{b_*} \sim 1/b_*$
3) Evolve again to connect Q_0 to μ_{b_*}
4) Place remaining NP parts in an exponent:

$$-g_{j/p}(x,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)$$

$$g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*;\mu) - \tilde{K}(b_\mathrm{T};\mu)$$

5) Ansatz for g-functions
6) Perform small-b_T expansions & drop $O(\Lambda_{QCD}\, b_{\max})$ errors

Conventional organization

$$\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$$

$$\int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\ e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\ \tilde{f}^{\mathrm{OPE}}_{j/p}(x_\mathrm{bj},\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*})\ \tilde{D}^{\mathrm{OPE}}_{h/j}(z_\mathrm{h},\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*})$$
$$\times\exp\left\{2\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1)-\ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right]+\ln\frac{Q^2}{\mu^2_{b_*}}\tilde{K}(b_*;\mu_{b_*})\right\}$$
$$\times\exp\left\{-g_{j/p}(x_\mathrm{bj},b_\mathrm{T})-g_{h/j}(z_\mathrm{h},b_\mathrm{T})-g_K(b_\mathrm{T})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}\ +O\left(b_\mathrm{max}\Lambda_\mathrm{QCD}\right)$$

$$\underbrace{\tilde{f}_{j/p}(x,b_T;\mu,\zeta)=\int_x^1\frac{\mathrm{d}\xi}{\xi}\tilde{C}_{j/k}(x/\xi,b_T;\zeta,\mu,\alpha_s(\mu))f_{k/p}(\xi;\mu)}_{\tilde{f}^{\mathrm{OPE}}_{j/p}(x,b_T;\mu,\zeta)}+O\left(b_T\Lambda_\mathrm{QCD}\right)$$

Steps:

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$$-g_{j/p}(x,b_\mathrm{T})\equiv\ln\left(\frac{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)$$

$$g_K(b_\mathrm{T})\equiv\tilde{K}(b_*;\mu)-\tilde{K}(b_\mathrm{T};\mu)$$

5) Ansatz for g-functions
6) Perform small-b_T expansions & drop $O(\Lambda_{QCD}\ b_{\max})$ errors

Check consistency

- $\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T})$
  is uniquely determined by its operator definition

- At $q_T \approx Q$

$$= \underbrace{\frac{1}{q_\mathrm{T}^2}\left[\mathcal{C}\left(q_\mathrm{T}/\mu_Q,\alpha_s\left(\mu_Q\right)\right)\otimes f_{j/p}(x;\mu_Q)\otimes d_{h/j}(z;\mu_Q)}_{q_T\sim Q,\, Q\to\infty \text{ asymptote}} + O\left(\frac{\Lambda_\mathrm{QCD}}{q_\mathrm{T}}\right)\right]$$

- For TMD pdfs & ffs

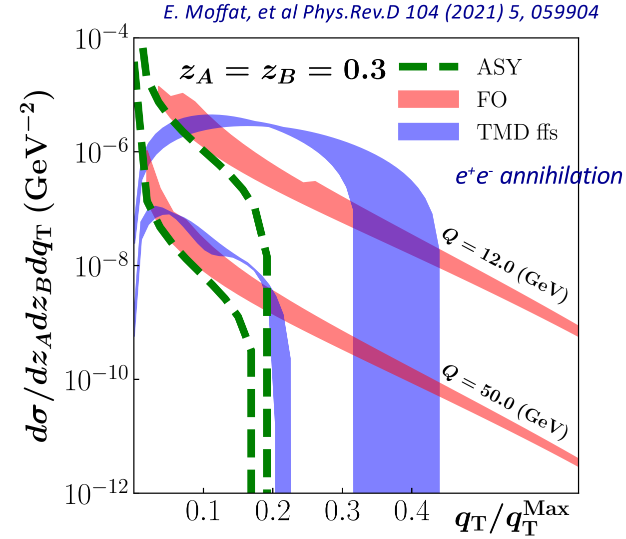
$$f_{j/p}(x,k_T\approx Q;\mu_Q,Q^2) = \frac{1}{k_T^2}\left[\int_x^1 \frac{\mathrm{d}\xi}{\xi} C_{j/k}(x/\xi,k_T/Q,\alpha_s(Q)) f_{k/p}(\xi;Q) + O\left(\frac{\Lambda_\mathrm{QCD}}{k_T}\right)\right]$$

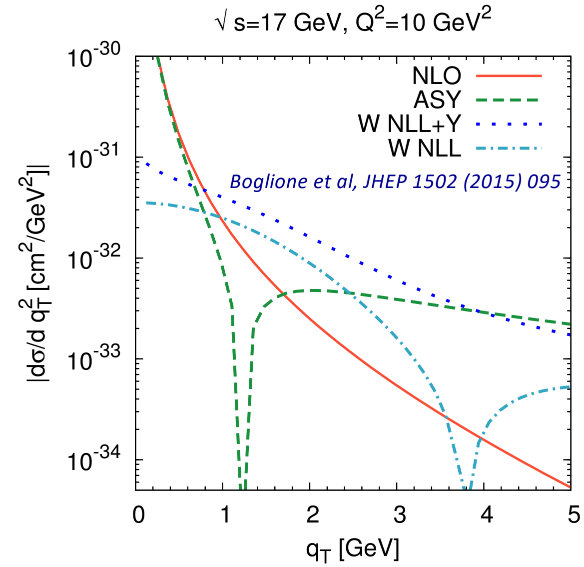
&

$$\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x,k_T;\mu_Q,Q^2)\approx f_{j/p}(x;\mu_Q)$$

- $\frac{\mathrm{d}}{\mathrm{d}b_\mathrm{max}}\left(\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{q}_\mathrm{T}\ldots}\right)=0$ *for* $b_\mathrm{max} \ll 1/\Lambda_{QCD}$

E. Moffat, et al Phys.Rev.D 104 (2021) 5, 059904



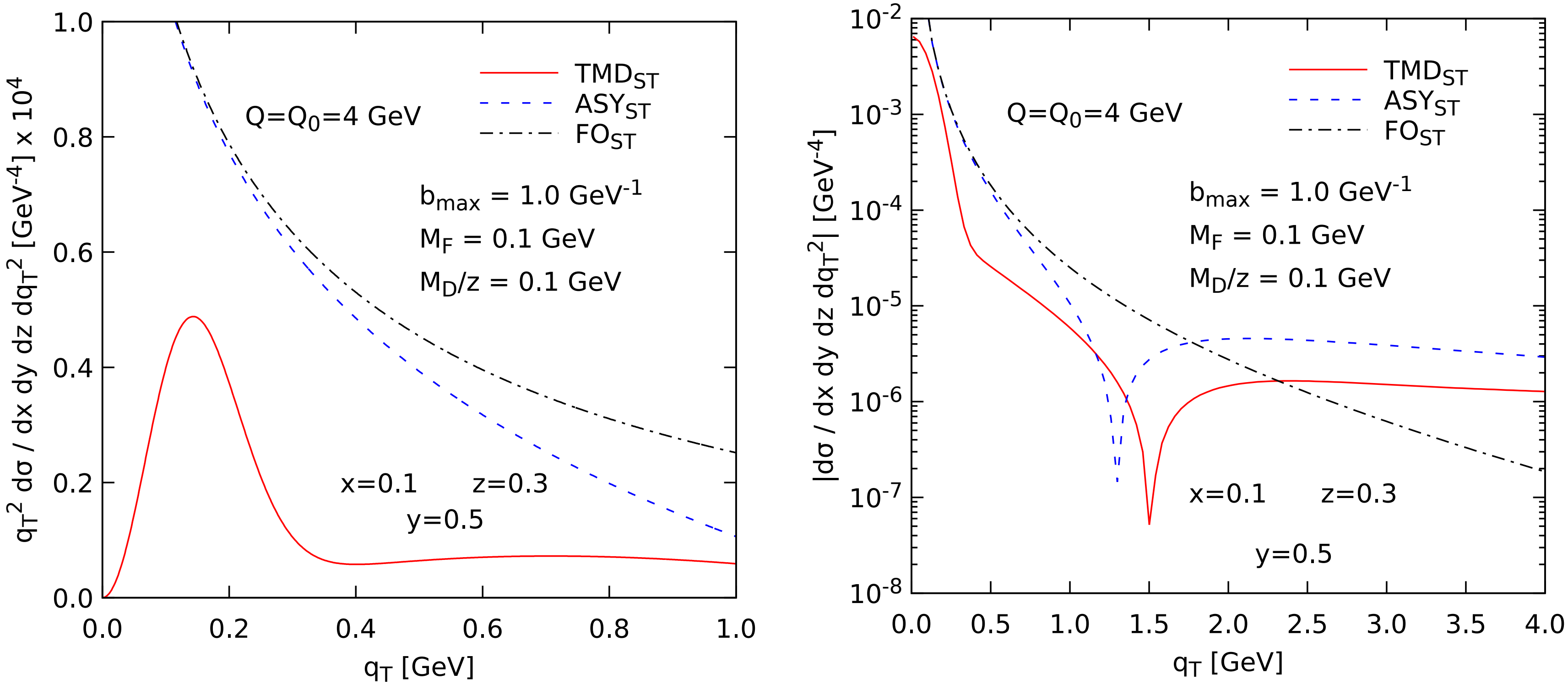


Conventional organization & problems

$$g_{j/p}(x, b_{\mathrm{T}}) = \frac{1}{4} M_F^2 \, b_{\mathrm{T}}^2$$

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4\, z^2} M_D^2 \, b_{\mathrm{T}}^2$$

(Example typical of conventional approach)



Conventional organization & problems

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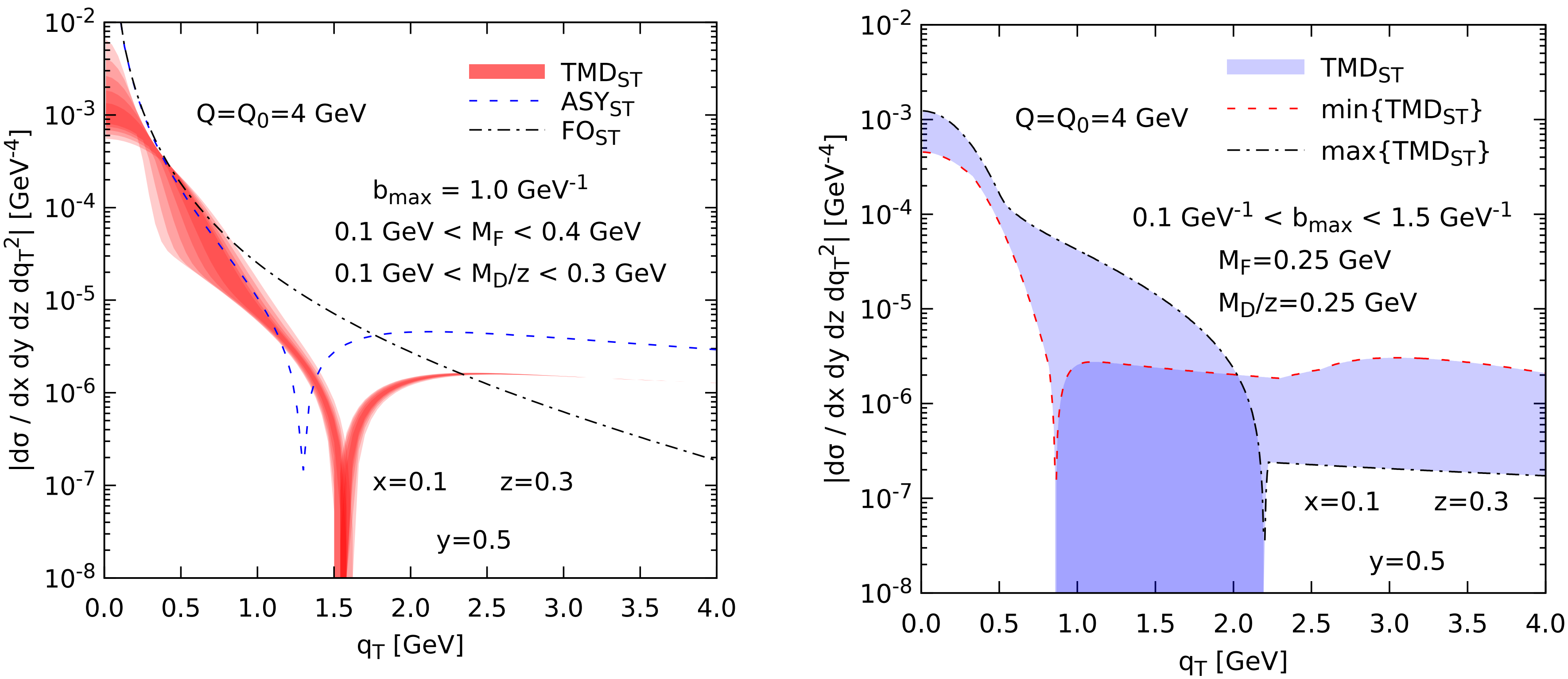


$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} = H(Q/\mu_Q)\underbrace{\int \mathrm{d}^2\mathbf{k}_{1T}\mathrm{d}^2\mathbf{k}_{2T} f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2) D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)}_{\textit{Well-defined operators for all TM}}\delta^{(2)}(\mathbf{q}_T+\mathbf{k}_{1T}-\mathbf{k}_{2T}) + \text{power suppressed}$$

Diagnosis

1) Consistency tests will generally fail for g-function ansatz unless constraints are imposed

2) No region at input scale $Q = Q_0$ where $\Lambda_{QCD} \ll q_T \ll Q_0$

3) Backwards evolution.
No large, perturbative $\ln \frac{Q_0}{q_T}$.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}^2\mathbf{q}_T\,\mathrm{d}x\,\mathrm{d}z} =$$

$$H(Q/\mu_Q)\int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\ \tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)$$

$$\times \exp\left\{\tilde{K}(b_\mathrm{T};\mu_{Q_0})\ln\left(\frac{Q^2}{Q_0^2}\right) + \int_{\mu_{Q_0}}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu');1) - \ln\frac{Q^2}{\mu'^2}\gamma_K(\alpha_s(\mu'))\right]\right\}$$

$$\tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = \tilde{D}_{\mathrm{inpt},h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{\overline{Q}_0},\overline{Q}_0^2)E(\overline{Q}_0/Q_0,b_\mathrm{T})$$

$$\tilde{f}_{i/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = \tilde{f}_{\mathrm{inpt},i/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{\overline{Q}_0},\overline{Q}_0^2)E(\overline{Q}_0/Q_0,b_\mathrm{T})$$

$$\lim_{b_\mathrm{T}\to 0}\overline{Q}_0 \sim 1/b_\mathrm{T}$$

Instead, characterize the full behavior of TMD functions at the input scale

A hadron structure oriented reorganization

1) Use the uniquely determined TMDs for all k_T

2) Smoothly transform between nonperturbative TM dependence at small TM ($k_T \approx \Lambda_{QCD}$) & perturbative (collinear) TM at large TM ($k_T \approx Q$)

3) (Approximate) probability interpretation

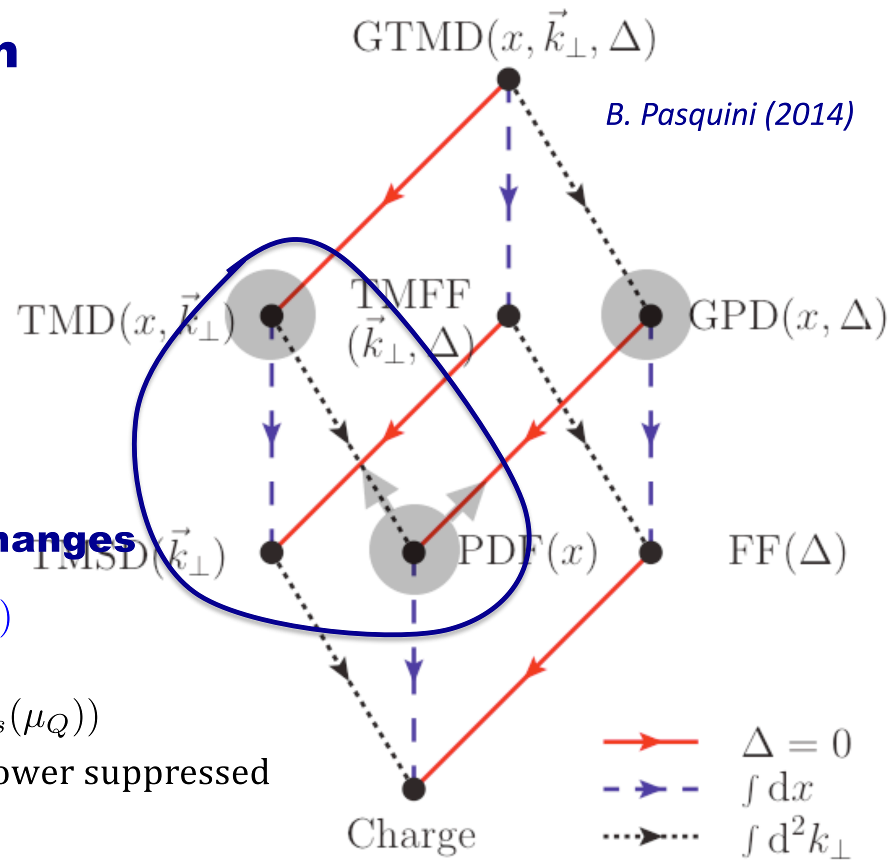
- Parton model: $\int \mathrm{d}^2\mathbf{k}_T f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q)$
- QCD: $\pi \int^{\mu_Q^2} \mathrm{d}k_T^2 f_{j/p}(x, k_T; \mu_Q, Q^2) = f_{j/p}(x; \mu_Q) + O(\alpha_s(\mu_Q))$

+ power suppressed

4) All should apply at input scale, Q_0

5) Pheno: Should be simple to swap one model/parametrization for another

Other changes



An $O(\alpha_s)$ example

$$f_{\text{inpt},i/p}(x,\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi}\frac{1}{k_\text{T}^2+m_{f_{i,p}}^2}\left[A_{i/p}^f(x;\mu_{Q_0}) + B_{i/p}^f(x;\mu_{Q_0})\ln\frac{Q_0^2}{k_\text{T}^2+m_{f_{i,p}}^2}\right] + \frac{1}{2\pi}\frac{1}{k_\text{T}^2+m_{f_{g,p}}^2}A_{i/p}^{f,g}(x;\mu_{Q_0}) + C_{i/p}^f\, f_{\text{core},i/p}(x,\boldsymbol{k}_\text{T};Q_0^2)$$

$$D_{\text{inpt},h/j}(z,z\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = \frac{1}{2\pi z^2}\frac{1}{k_\text{T}^2+m_{D_{h,j}}^2}\left[A_{h/j}^D(z;\mu_{Q_0}) + B_{h/j}^D(z;\mu_{Q_0})\ln\frac{Q_0^2}{k_\text{T}^2+m_{D_{h,j}}^2}\right] + \frac{1}{2\pi z^2}\frac{1}{k_\text{T}^2+m_{D_{h,g}}^2}A_{h/j}^{D,g}(z;\mu_{Q_0}) + C_{h/j}^D\, D_{\text{core},h/j}(z,z\boldsymbol{k}_\text{T};Q_0^2)$$

- C^f & C^D constrained by:

$$f_{i/p}^c(x;\mu_{Q_0}) \equiv 2\pi\int_0^{\mu_{Q_0}} \mathrm{d}k_\text{T}\,k_\text{T} f_{i/p}(x,\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = f_{i/p}(x;\mu_{Q_0}) + \mathcal{H}_{i/i'}\otimes f_{i'/p} + \text{p.s.}$$

$$d_{h/j}^c(z;\mu_{Q_0}) \equiv 2\pi z^2\int_0^{\mu_{Q_0}} \mathrm{d}k_\text{T}\,k_\text{T} D_{h/j}(z,z\boldsymbol{k}_\text{T};\mu_{Q_0},Q_0^2) = d_{h/j}(z;\mu_{Q_0}) + \mathcal{H}_{j'/j}\otimes d_{h/j'} + \text{p.s.}$$

An $O(\alpha_s)$ example

- Parametrizing the very small transverse momentum

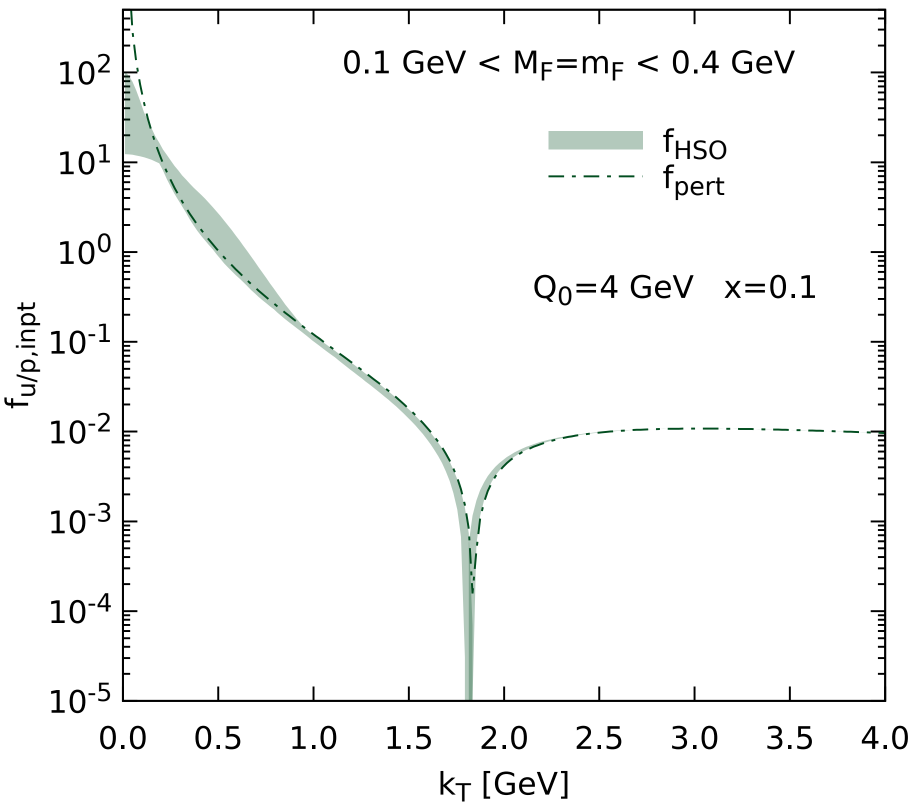
  – Gaussian model

$$f^{\text{Gauss}}_{\text{core},i/p}(x, \boldsymbol{k}_{\text{T}}; Q_0^2) = \frac{e^{-k_{\text{T}}^2/M_{\text{F}}^2}}{\pi M_{\text{F}}^2}\,, \qquad D^{\text{Gauss}}_{\text{core},h/j}(z, z\boldsymbol{k}_{\text{T}}; Q_0^2) = \frac{e^{-z^2\, k_{\text{T}}^2/M_{\text{D}}^2}}{\pi M_{\text{D}}^2}$$

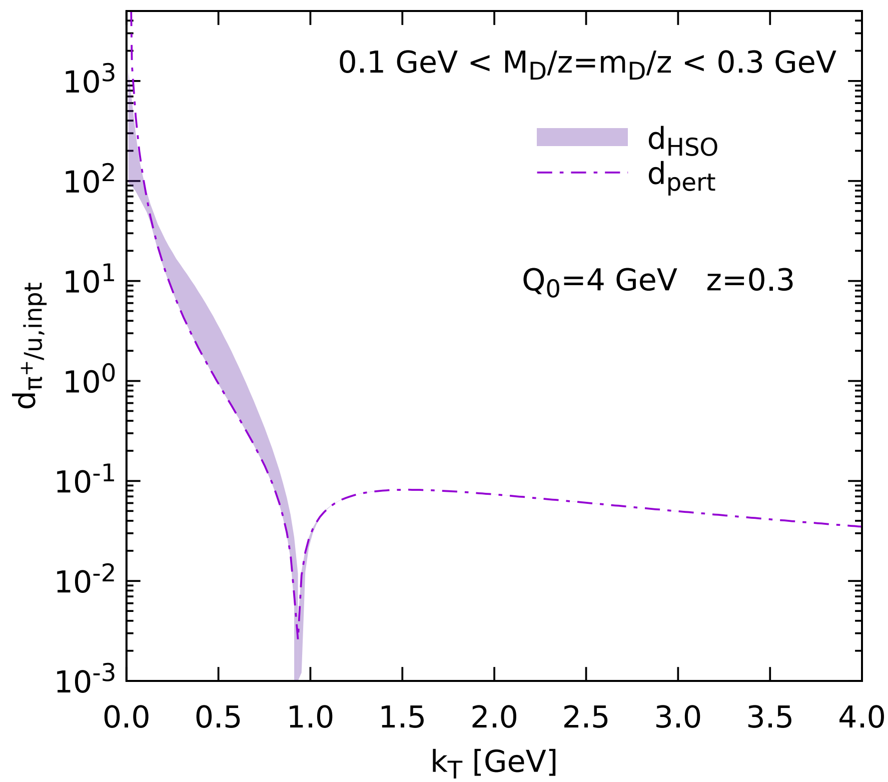
  – Spectator model

$$f^{\text{Spect}}_{\text{core},i/p}(x, \boldsymbol{k}_{\text{T}}; Q_0^2) = \frac{6 M_{0\text{F}}^6}{\pi\,(2M_{\text{F}}^2 + M_{0\text{F}}^2)}\,\frac{M_{\text{F}}^2 + k_{\text{T}}^2}{(M_{0\text{F}}^2 + k_{\text{T}}^2)^{\,4}}\quad , \quad D^{\text{Spect}}_{\text{core},h/j}(z, z\boldsymbol{k}_{\text{T}}; Q_0^2) = \frac{2 M_{0\text{D}}^4}{\pi\,(M_{\text{D}}^2 + M_{0\text{D}}^2)}\,\frac{M_{\text{D}}^2 + k_{\text{T}}^2 z^2}{(M_{0\text{D}}^2 + k_{\text{T}}^2 z^2)^{\,3}}$$

up-quark TMD pdfs

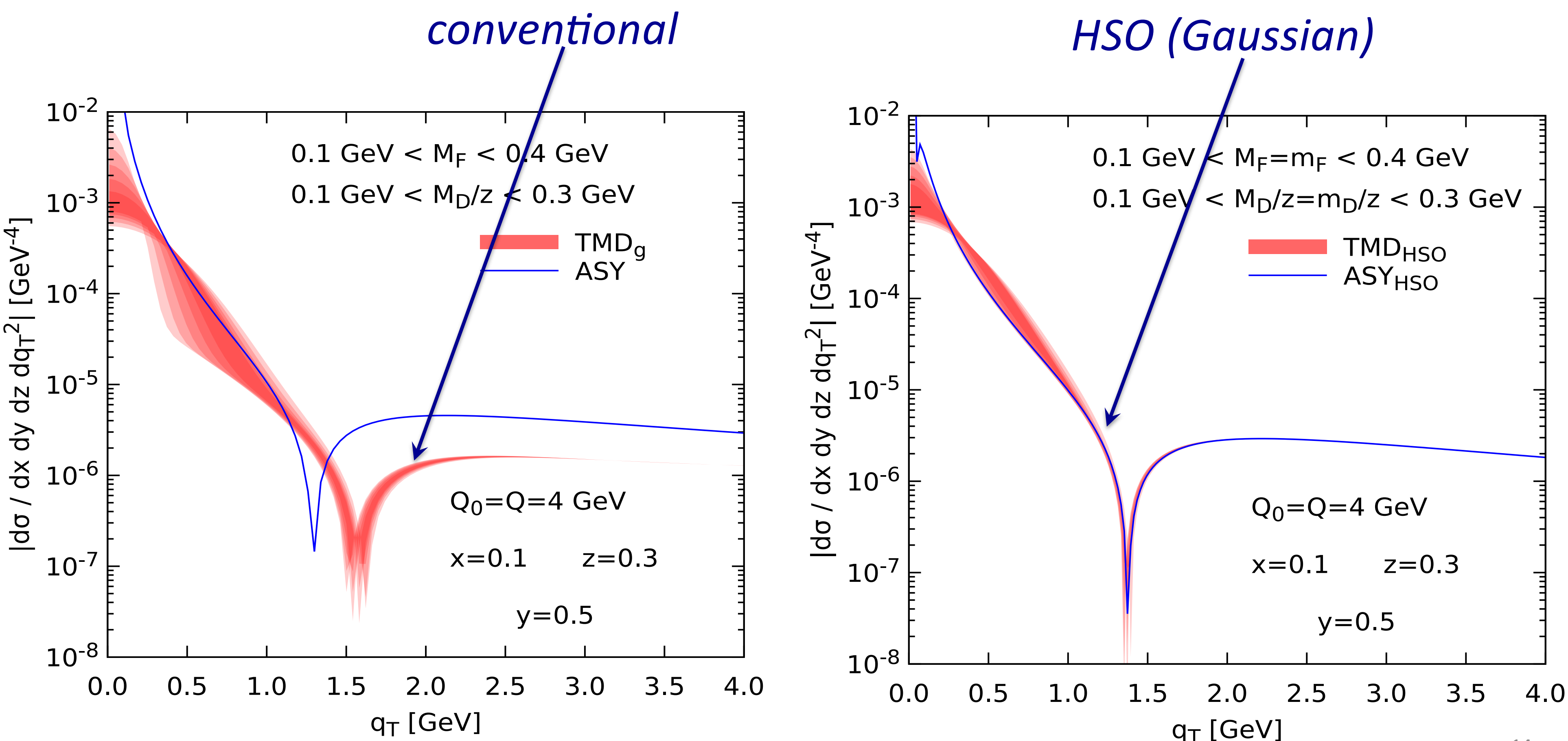


π^+ from up-quark TMD ff

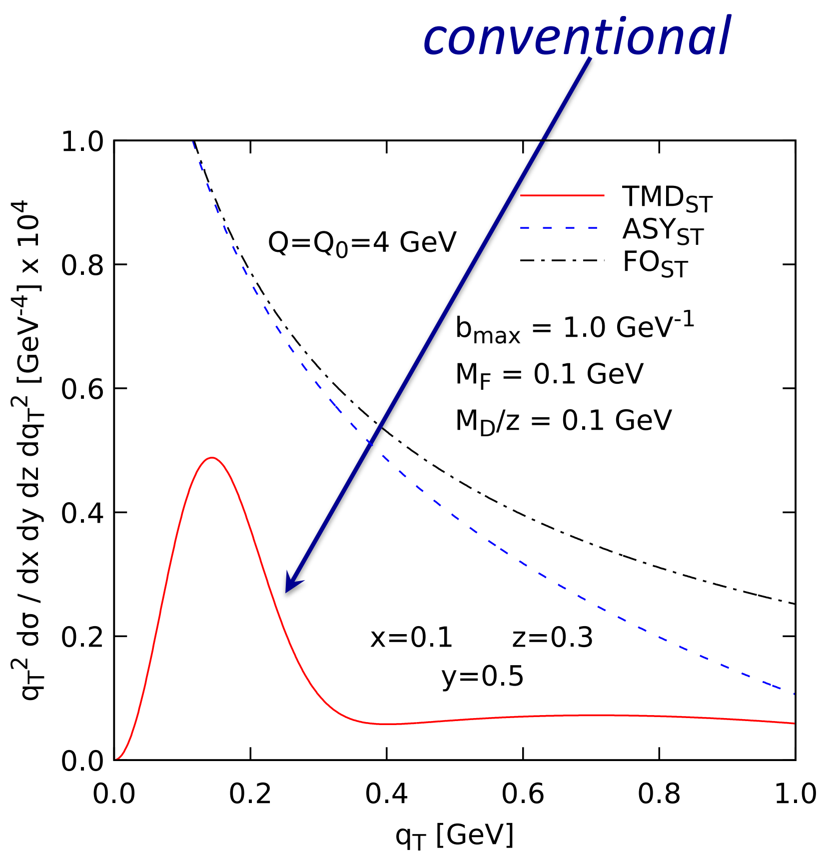


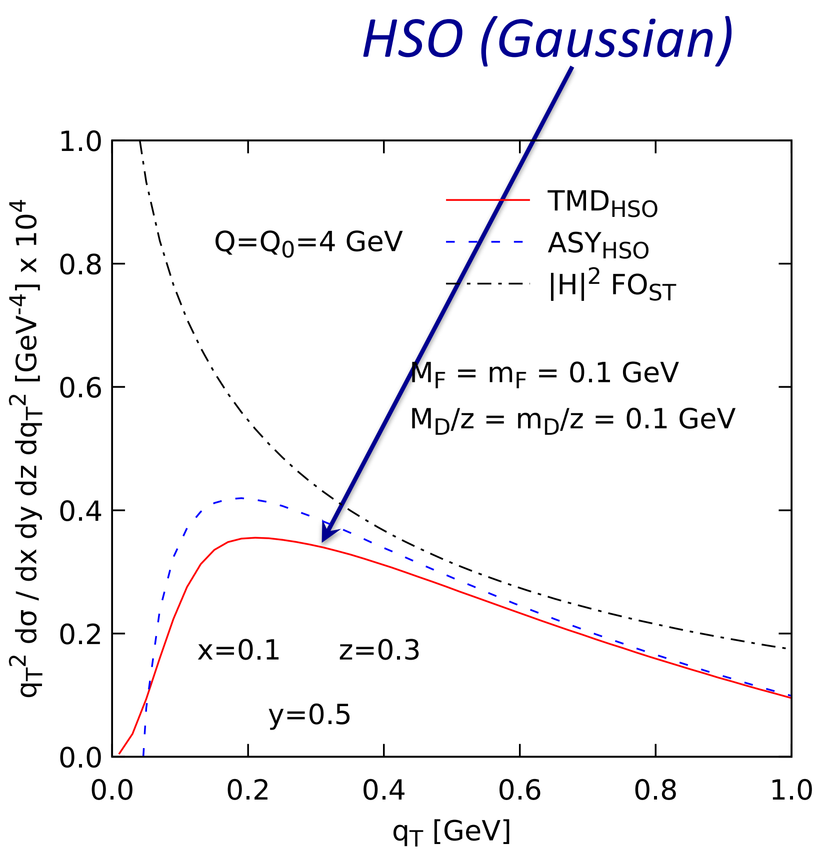
HSO (Gaussian)

Compare standard with constrained



Compare standard with constrained





Summary

- Switching to a hadron structure oriented approach to phenomenological implementations of TMD factorization improves consistency in the large transverse behavior of TMD correlation functions

- Necessary for understanding the shapes of nonperturbative components of distributions, separating perturbative and nonpeturbative

- Next steps:

  - Applications
  - Higher orders
  - Incorporating NP calculations (lattice, EFTs, models etc)

Why “hadron structure oriented?”

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