QCD EVOLUTION
2024
Pavia

A first implementation of the HSO approach to TMD phenomenology

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Based on:

JOGH, T.C. Rogers T., N. Sato
Phys.Rev.D 106 (2022) 3, 034002 • e-Print: 2205.05750 [hep-ph]

JOGH, T. Rainaldi, T.C. Rogers
e-Print: 2303.04921 [hep-ph]
Accepted in Phys. Rev. D

F. Aslan, M. Boglione, JOGH, T.C. Rogers, T. Rainaldi, A. Simonelli
e-Print: 2401.14266 [hep-ph]

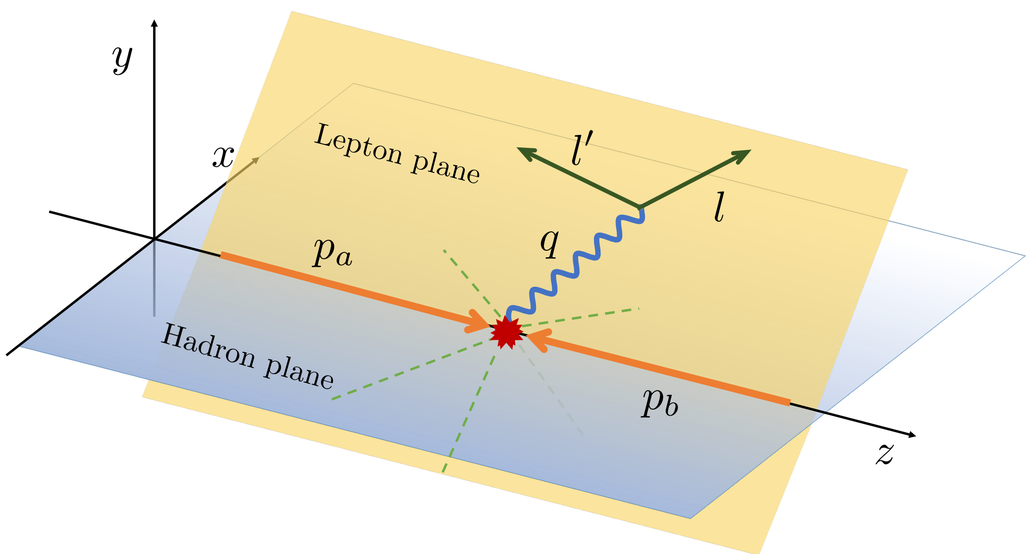
OUTLINE

- ✳ Building models in HSO
- ✳ HSO Strategy
- ✳ DY and Z^0 production examples

"Hadron Structure Oriented approach"

✳ Building models in HSO

Unpolarized DY cross section (TMD region)



$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_{h\mathrm{T}}^2\,\mathrm{d}Q^2\,\mathrm{d}y_h} = \frac{2\pi^2\alpha_{\mathrm{em}}^2}{3sQ^2}\left(2F_{UU}^1 + \cancel{F_{UU}^2}\right) .$$

$$q_h = \left(e^{y_h}\sqrt{\frac{Q^2+q_{h\mathrm{T}}^2}{2}}, e^{-y_h}\sqrt{\frac{Q^2+q_{h\mathrm{T}}^2}{2}}, \boldsymbol{q}_{h\mathrm{T}}\right)$$

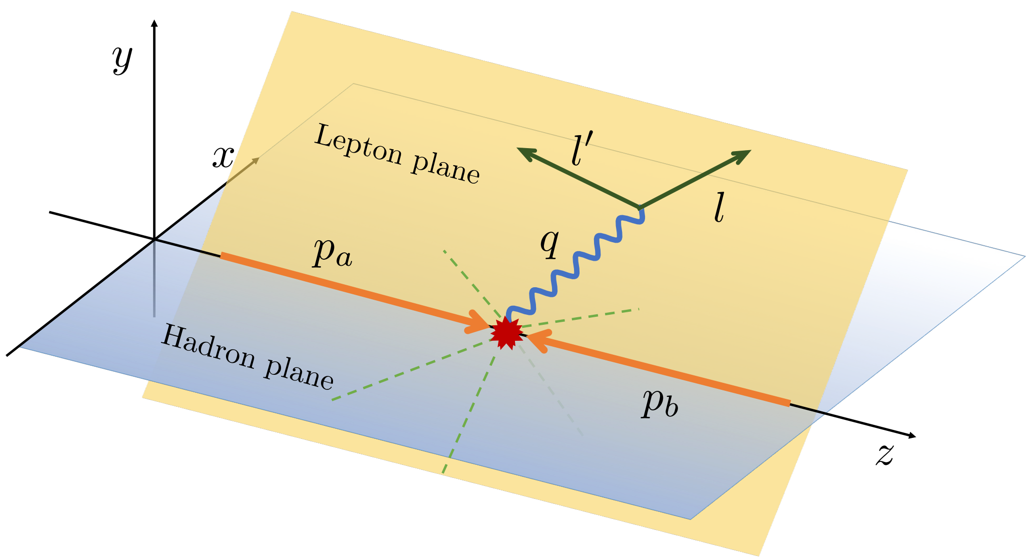
$$x_a = \frac{Qe^{y_h}}{\sqrt{s\left(1+\frac{q_{\mathrm{T}}^2}{Q^2}\right)}}, \qquad x_b = \frac{Qe^{-y_h}}{\sqrt{s\left(1+\frac{q_{\mathrm{T}}^2}{Q^2}\right)}} .$$

$$F_{UU}^1 = \sum_j e_j^2 \frac{|H_{j\bar{j}}|^2}{4\pi^2 N_c}\int \mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}\, e^{i\boldsymbol{q}_{h\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}\,\tilde{f}_{j/h_a}(x_a,\boldsymbol{b}_{\mathrm{T}};\mu_Q,Q^2)\;\tilde{f}_{\bar{j}/h_b}(x_b,\boldsymbol{b}_{\mathrm{T}};\mu_Q,Q^2) + (a \longleftrightarrow b)$$

$$+\boxed{O\left(m/Q, q_{\mathrm{T}}/Q\right)}$$

errors

Unpolarized DY cross section (TMD region)



CS kernel

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_{\mathrm{T}}; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \boxed{\tilde{K}(b_{\mathrm{T}}; \mu)\,,}$$

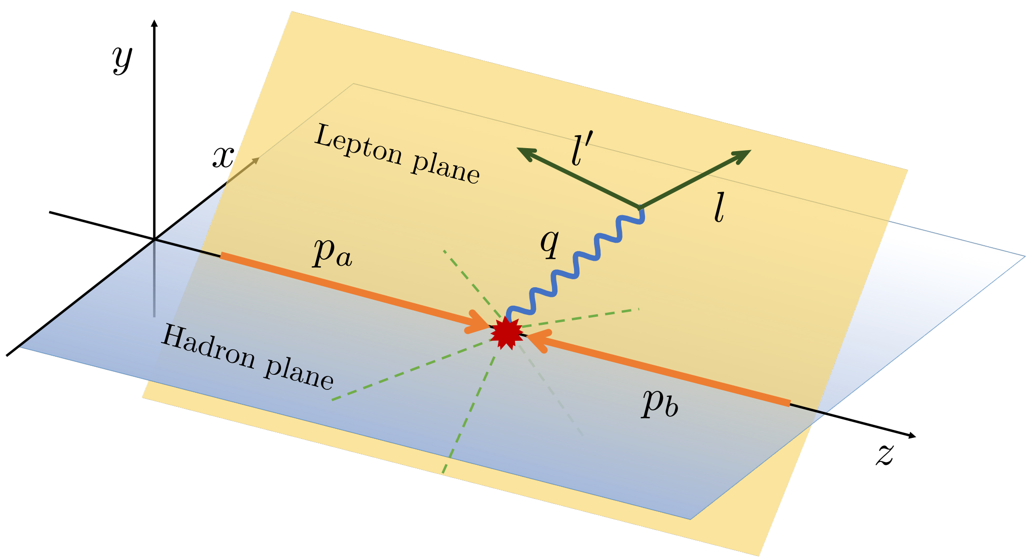
$$\frac{\mathrm{d}\tilde{K}(b_{\mathrm{T}}; \mu)}{\mathrm{d}\ln \mu} = -\gamma_K(\alpha_s(\mu))\,,$$

$$\frac{\mathrm{d}\ln \tilde{f}_{j/p}(x, b_{\mathrm{T}}; \mu, \zeta)}{\mathrm{d}\ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

$$F_{UU}^{1} = \sum_j e_j^2 \frac{|H_{j\bar{j}}|^2}{4\pi^2 N_c} \int \mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}}\, e^{i\boldsymbol{q}_{h\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}}\, \tilde{f}_{j/h_a}(x_a, \boldsymbol{b}_{\mathrm{T}}; \mu_Q, Q^2)\ \tilde{f}_{\bar{j}/h_b}(x_b, \boldsymbol{b}_{\mathrm{T}}; \mu_Q, Q^2) + (a \longleftrightarrow b)$$

$$+ O\left(m/Q, q_{\mathrm{T}}/Q\right)$$

Unpolarized DY cross section (TMD region)



CS kernel

$$\frac{\partial \ln \tilde{f}_{j/p}(x, b_{\mathrm{T}}; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \boxed{\tilde{K}(b_{\mathrm{T}}; \mu)\,,}$$

$$\frac{\mathrm{d}\tilde{K}(b_{\mathrm{T}}; \mu)}{\mathrm{d}\ln \mu} = -\gamma_K(\alpha_s(\mu))\,,$$

$$\frac{\mathrm{d}\ln \tilde{f}_{j/p}(x, b_{\mathrm{T}}; \mu, \zeta)}{\mathrm{d}\ln \mu} = \gamma(\alpha_s(\mu); \zeta/\mu^2)$$

Solve evolution equations and write in terms of **input scale**

$$F_{UU}^1 = \sum_j e_j^2 \frac{|H_{j\bar{j}}|^2}{4\pi^2 N_c} \int \mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}\, e^{i\boldsymbol{q}_{h\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}\, \tilde{f}_{j/h_a}(x_a, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)\; \tilde{f}_{\bar{j}/h_b}(x_b, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)\times$$

$$\times \exp\left\{\tilde{K}(b_{\mathrm{T}}; \mu_{Q_0}) \ln\left(\frac{Q^2}{Q_0^2}\right) + \int_{\mu_{Q_0}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma(\alpha_s(\mu'); 1) - \ln\left(\frac{Q^2}{\mu'^2}\right)\gamma_K(\alpha_s(\mu'))\right]\right\} + (a \longleftrightarrow b)$$

Unpolarized DY cross section (TMD region)

Usually, here one rearranges the expression to take advantage of the **small-b_T OPE.** We depart from this, but one can see a correspondence with the usual treatment (later).

$$
\begin{aligned}
F_{UU}^{1} = & \sum_{j} e_j^2 \frac{\left|H_{j\bar{j}}\right|^2}{4\pi^2 N_c} \int \mathrm{d}^2 \boldsymbol{b}_{\mathrm{T}} \, e^{i \boldsymbol{q}_{h\mathrm{T}} \cdot \boldsymbol{b}_{\mathrm{T}}} \, \tilde{f}_{j/h_a}(x_a, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2) \; \tilde{f}_{\bar{j}/h_b}(x_b, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2) \times \\
& \times \exp\left\{ \tilde{K}(b_{\mathrm{T}}; \mu_{Q_0}) \ln\left(\frac{Q^2}{Q_0^2}\right) + \int_{\mu_{Q_0}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln\left(\frac{Q^2}{{\mu'}^2}\right) \gamma_K(\alpha_s(\mu')) \right] \right\} + (a \longleftrightarrow b)
\end{aligned}
$$

We build models in transverse momentum space.

$$f_{\mathrm{operator}}(x, k_{\mathrm{T}}; \mu_{Q_0}, \boxed{Q_0^2}) \Longrightarrow f_{\mathrm{inpt}}(x, k_{\mathrm{T}}; \mu_{Q_0}, \boxed{Q_0^2})$$

input scale

Abstract　　　　Pheno

Special role of input scale:

- Larger values: factorization/pQCD works better
- Small values: more prominent intrinsic kT

We build models in transverse momentum space.

$$f_{\text{operator}}(x, k_{\text{T}}; \mu_{Q_0}, \boxed{Q_0^2}) \Longrightarrow f_{\text{inpt}}(x, k_{\text{T}}; \mu_{Q_0}, \boxed{Q_0^2})$$

input
scale

Abstract Pheno

Must preserve fundamental properties of the operator definition in our models at the **input scale.**

We do it additively (other options allowed)

$$f_{\mathrm{inpt},i/p}(x, k_{\mathrm{T}}; \mu_{Q_0}, Q_0^2) =$$

$$C_{i/p}\, f_{\mathrm{core},i/p}(x, k_{\mathrm{T}}; Q_0^2) \quad +$$

Start with a “core” model/parametrization for intrinsic k_{T}

We do it additively (other options allowed)

$$f_{\text{inpt},i/p}(x, k_{\text{T}}; \mu_{Q_0}, Q_0^2) =$$

$$C_{i/p}\, f_{\text{core},i/p}(x, k_{\text{T}}; Q_0^2) \quad +$$

Make sure the model has the large k_T behavior of the TMD in the $k_T \sim Q_0$ approximation

$$f_{i/p}^{\text{operator}}(x, k_{\text{T}} \sim Q_0; \mu_{Q_0}, Q_0^2) = f_{i/p}^{\text{pert}}(x, k_{\text{T}}; \mu_{Q_0}, Q_0^2)$$

$$= \boxed{\frac{1}{2\pi}\frac{1}{k_{\text{T}}^2}\left[A_{i/p}(x;\mu_{Q_0}) + B_{i/p}(x;\mu_{Q_0}) \ln\left(\frac{Q_0^2}{k_{\text{T}}^2}\right) + A_{i/p}^g(x;\mu_{Q_0})\right]}$$

pQCD tail, related to OPE in b_T space

We do it additively (other options allowed)

model masses

$$f_{\text{inpt},i/p}(x,k_T;\mu_{Q_0},Q_0^2) =$$

$$C_{i/p}\, f_{\text{core},i/p}(x,k_T;Q_0^2) \;+\; \frac{1}{2\pi}\frac{1}{k_T^2+\boxed{m_{i,p,A}^2}}A_{i/p}(x;\mu_{Q_0})$$

$$+\; \frac{1}{2\pi}\frac{1}{k_T^2+\boxed{m_{i,p,B}^2}}B_{i/p}(x;\mu_{Q_0})\ln\left(\frac{Q_0^2}{k_T^2+\boxed{m_{i,p,L}^2}}\right) \;+\; \frac{1}{2\pi}\frac{1}{k_T^2+\boxed{m_{g,p}^2}}A^g_{i/p}(x;\mu_{Q_0})$$

Make sure the model has the large k_T behavior of the TMD in the $k_T \sim Q_0$ approximation

$$f_{i/p}^{\text{operator}}(x,k_T\sim Q_0;\mu_{Q_0},Q_0^2) = f_{i/p}^{\text{pert}}(x,k_T;\mu_{Q_0},Q_0^2)$$

$$= \frac{1}{2\pi}\frac{1}{k_T^2}\left[A_{i/p}(x;\mu_{Q_0}) + B_{i/p}(x;\mu_{Q_0})\ln\left(\frac{Q_0^2}{k_T^2}\right) + A^g_{i/p}(x;\mu_{Q_0})\right]$$

We do it additively (other options allowed)

$$f_{\mathrm{inpt},i/p}(x,k_{\mathrm{T}};\mu_{Q_0},Q_0^2) =$$

$$\boxed{C_{i/p}}\, f_{\mathrm{core},i/p}(x,k_{\mathrm{T}};Q_0^2) \;+\; \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{i,p,A}^2}A_{i/p}(x;\mu_{Q_0})$$
$$+\; \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{i,p,B}^2}B_{i/p}(x;\mu_{Q_0})\ln\left(\frac{Q_0^2}{k_{\mathrm{T}}^2+m_{i,p,L}^2}\right) \;+\; \frac{1}{2\pi}\frac{1}{k_{\mathrm{T}}^2+m_{g,p}^2}A_{i/p}^{g}(x;\mu_{Q_0})$$

Impose the QCD integral relation

$$2\pi\int_0^{\mu_{Q_0}} \mathrm{d}k_{\mathrm{T}}\, k_{\mathrm{T}} f_{i/p}^{\mathrm{operator}}(x,\boldsymbol{k}_{\mathrm{T}};\mu_{Q_0},\mu_{Q_0}^2) \;=\; f_{i/p}^{\overline{\mathrm{MS}}}(x;\mu_{Q_0}) + \Delta_{i/p}(\alpha_s(\mu_{Q_0})) + O\left(\frac{m^2}{\mu_{Q_0}^2}\right)$$

Determines the normalization of the core function

We do it additively (other options allowed)

In b_T space

Bessel "K"

$$\tilde{f}_{\mathrm{inpt},j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = K_0\,(m_{i,p}b_\mathrm{T})\,A_{i/p}(x;\mu_{Q_0}) + K_0\,(m_{i,p}b_\mathrm{T})\ln\left(\frac{Q_0^2 b_\mathrm{T}}{2m_{i,p}e^{-\gamma_E}}\right)B_{i/p}(x;\mu_{Q_0})$$
$$+ K_0\,(m_{g,p}b_\mathrm{T})\,A^g_{i/p}(x;\mu_{Q_0}) + C_{i/p}\,\tilde{f}_{\mathrm{core},i/p}(x,b_\mathrm{T};Q_0^2)\,,$$

$$\tilde{f}_{\mathrm{inpt},j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) \to \tilde{f}_{\mathrm{OPE},j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) \qquad (b_\mathrm{T}\to 0)$$

We do it additively (other options allowed)

In b_T space   Bessel "K"

$$\tilde{f}_{\mathrm{inpt},j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) = \boxed{K_0}\,(m_{i,p}b_\mathrm{T})\,A_{i/p}(x;\mu_{Q_0}) + K_0\,(m_{i,p}b_\mathrm{T})\ln\left(\frac{Q_0^2 b_\mathrm{T}}{2m_{i,p}e^{-\gamma_E}}\right)B_{i/p}(x;\mu_{Q_0})$$
$$+ \boxed{K_0}\,(m_{g,p}b_\mathrm{T})\,A^g_{i/p}(x;\mu_{Q_0}) + C_{i/p}\,\tilde{f}_{\mathrm{core},i/p}(x,b_\mathrm{T};Q_0^2)\,,$$

$$\tilde{f}_{\mathrm{inpt},j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) \to \tilde{f}_{\mathrm{OPE},j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2) \qquad (b_\mathrm{T}\to 0)$$

Note, it does not necessarily work the other way around: starting from the OPE and multiplying by a model does not guarantee the constraints to hold.

We do it additively (other options allowed)

$$f_{\text{inpt},i/p}(x, k_{\mathrm{T}}; \mu_{Q_0}, Q_0^2) =$$

$$C_{i/p}\, f_{\text{core},i/p}(x, k_{\mathrm{T}}; Q_0^2) \;+\; \frac{1}{2\pi} \frac{1}{k_{\mathrm{T}}^2 + m_{i,p,A}^2} A_{i/p}(x; \mu_{Q_0})$$

$$+\; \frac{1}{2\pi} \frac{1}{k_{\mathrm{T}}^2 + m_{i,p,B}^2} B_{i/p}(x; \mu_{Q_0}) \ln\left(\frac{Q_0^2}{k_{\mathrm{T}}^2 + m_{i,p,L}^2}\right) \;+\; \frac{1}{2\pi} \frac{1}{k_{\mathrm{T}}^2 + m_{g,p}^2} A_{i/p}^{g}(x; \mu_{Q_0})$$

In b_{T} space

$$\tilde{f}_{\text{inpt},j/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2) = \boxed{K_0} (m_{i,p} b_{\mathrm{T}})\, A_{i/p}(x; \mu_{Q_0}) + K_0 (m_{i,p} b_{\mathrm{T}}) \ln\left(\frac{Q_0^2 b_{\mathrm{T}}}{2 m_{i,p} e^{-\gamma_E}}\right) B_{i/p}(x; \mu_{Q_0})$$
$$+ \boxed{K_0} (m_{g,p} b_{\mathrm{T}})\, A_{i/p}^{g}(x; \mu_{Q_0}) + C_{i/p}\, \tilde{f}_{\text{core},i/p}(x, b_{\mathrm{T}}; Q_0^2)\,,$$

Bessel “K”

SIMILAR STEPS FOR THE KERNEL

Model in the HSO approach: CS kernel

$$D_K(\mu_{Q_0}) = -b_K + \frac{2\alpha_s(\mu_{Q_0})C_F}{\pi} \ln\left(\frac{m_K}{\mu_{Q_0}}\right)$$

$$K_{\rm inpt}(k_{\rm T};\mu_{Q_0}) = A_K^{(1)}(\mu_{Q_0})\frac{1}{k_{\rm T}^2+m_K^2} + K_{\rm core}(k_{\rm T}) + D_K(\mu_{Q_0})\delta^{(2)}(\boldsymbol{k}_{\rm T})$$

$$K^{(1)}(k_{\rm T};\mu_{Q_0}) = \frac{\alpha_s(\mu_{Q_0})C_F}{\pi^2}\frac{1}{k_{\rm T}^2}\,.$$

Perturbative tail

$$\frac{{\rm d}\tilde{K}^{(n)}_{\rm input}(b_{\rm T};\mu)}{{\rm d}\ln\mu} = -\gamma_K^{(n)}(\alpha_s(\mu)) + O\left(\alpha_s(\mu)^{n+1}\right)$$

Evolution equation valid
Up to higher corrections

Expressions useful for pheno at Q ≈ Q_0

✳HSO Strategy

HSO Strategy.

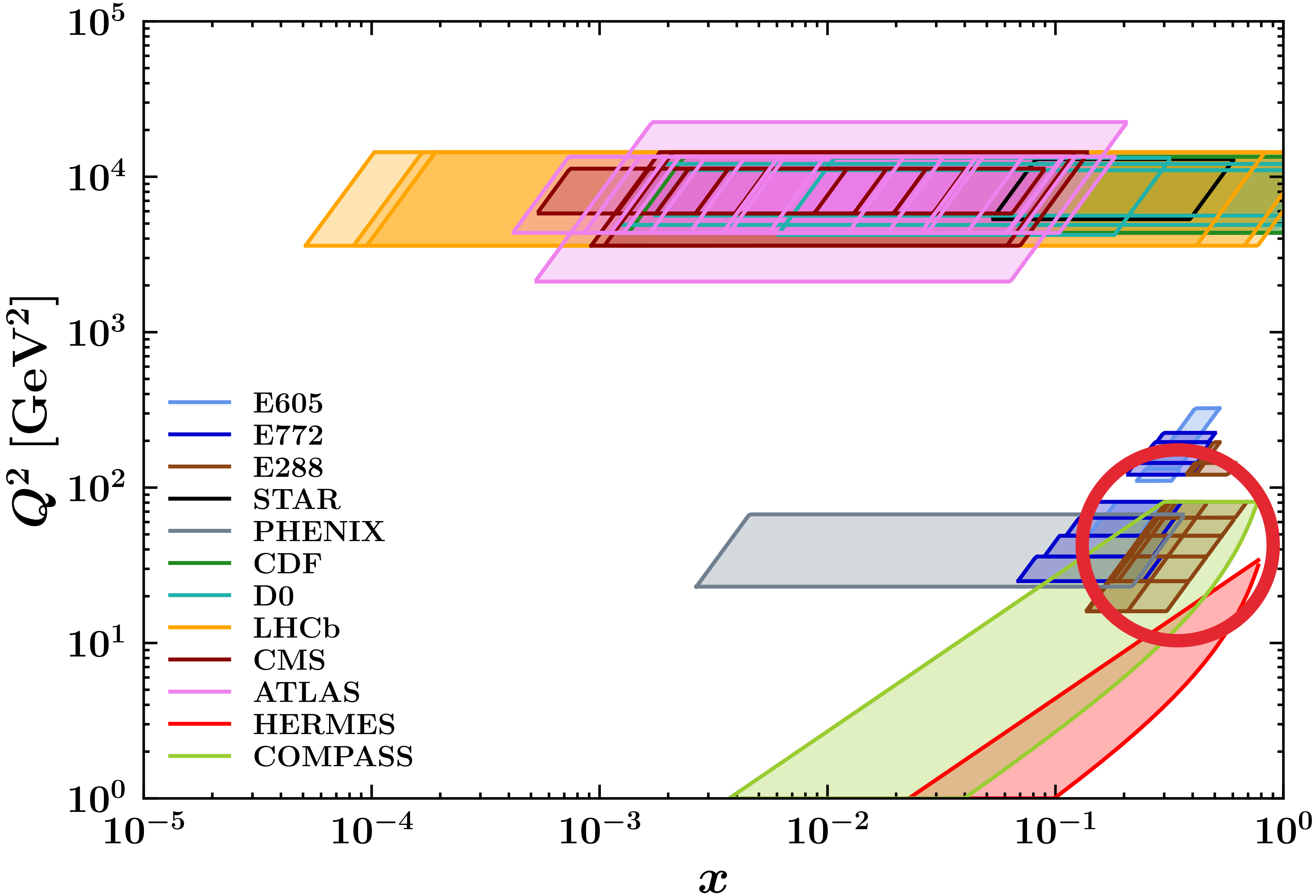
-Use theoretical constraints, don't trust the fit will do this job by itself.

-Check/improve constraints

-Prioritize the role of lower scale data (more information about intrinsic kT)

-Emphasize the predictive aspect of factorization theorems

Emphasize the predictive aspect of factorization theorems



POSTDICTIONS

(NO FITTING)

CDF II

Fit

E288, E605

(for now)

Plot from (MAP collaboration):
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✳ DY and Z^0 production examples

Simple minimization procure

Nuisance parameter for normalization uncertainty.

$$\chi^2 = \frac{(1-N)^2}{\delta_N^2} + \sum_i \frac{(d_i - t_i/N)^2}{\sigma_i^2}\,,$$

(Produce errors with eigensets)

Fit only $q_{\mathrm{T}} \leq 0.2\,Q\,,$

Simple treatment of target

$$f_{i/t} = \frac{Z}{A}\,f_{i/p} + \frac{A-Z}{A}\,f_{i/n}\,,$$

Example I: fit E288 (only) vs fit E605 (only)

Models for core functions

$$f^{\text{Gauss}}_{\text{core},i/p}(x, \boldsymbol{k}_{\text{T}}; Q_0^2) = \frac{e^{-k_{\text{T}}^2/M_{\text{F}}^2}}{\pi M_{\text{F}}^2}$$

$$K_{\text{core}}(k_{\text{T}}) = \frac{b_K}{4\pi m_K^2} e^{-\frac{k_{\text{T}}^2}{4m_K^2}}$$

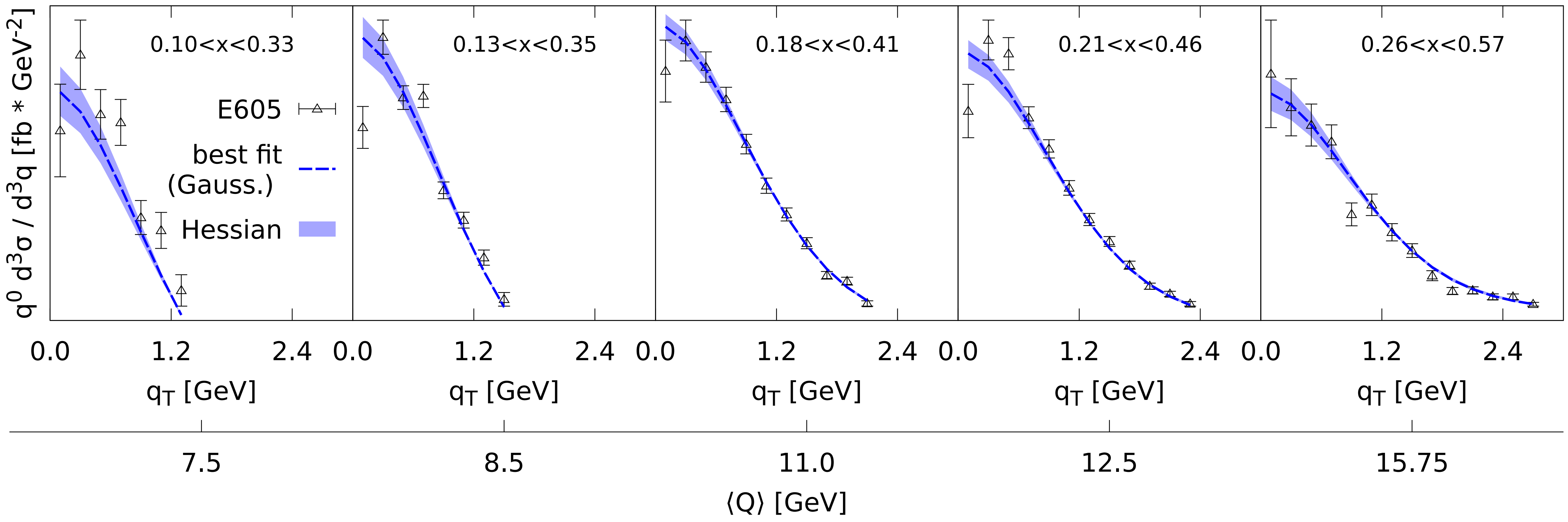
$$M_{\text{F}} \to M_0 + M_1 \log(1/x)\,,$$

Free parameters M_0, M_1 , b_k

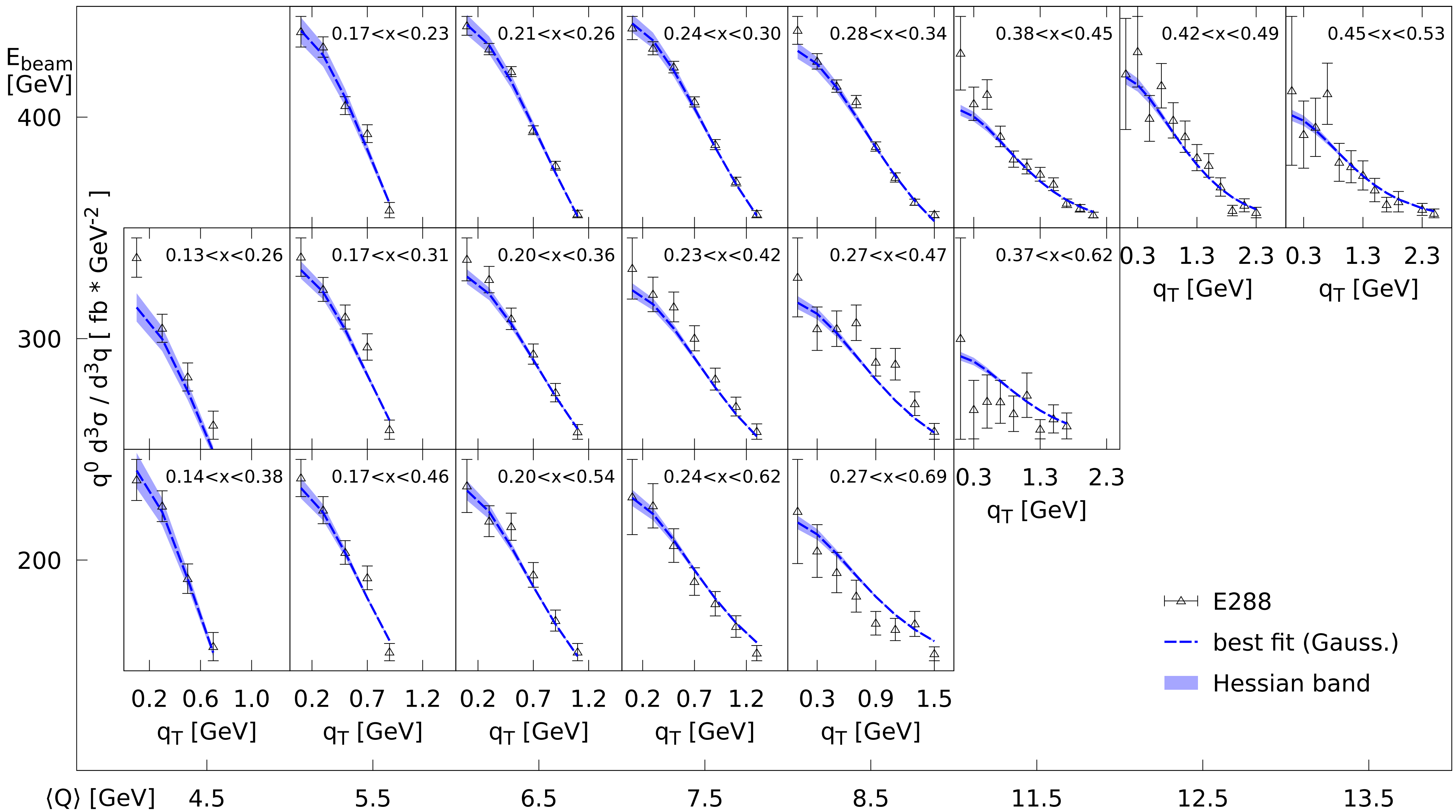
Other small model
masses fixed to $0.3\,\text{GeV}$

Example I: fit E288 (only) vs fit E605 (only)

Gaussian fits		
	E288 (130 pts.)	E605 (52 pts.)
$\chi^2_{\rm dof}$	1.04	1.68
M_0 (GeV)	0.0576	0.404
M_1 (GeV)	0.403	0.290
b_K	2.12	0.744
N(nuisance)	1.29	1.28

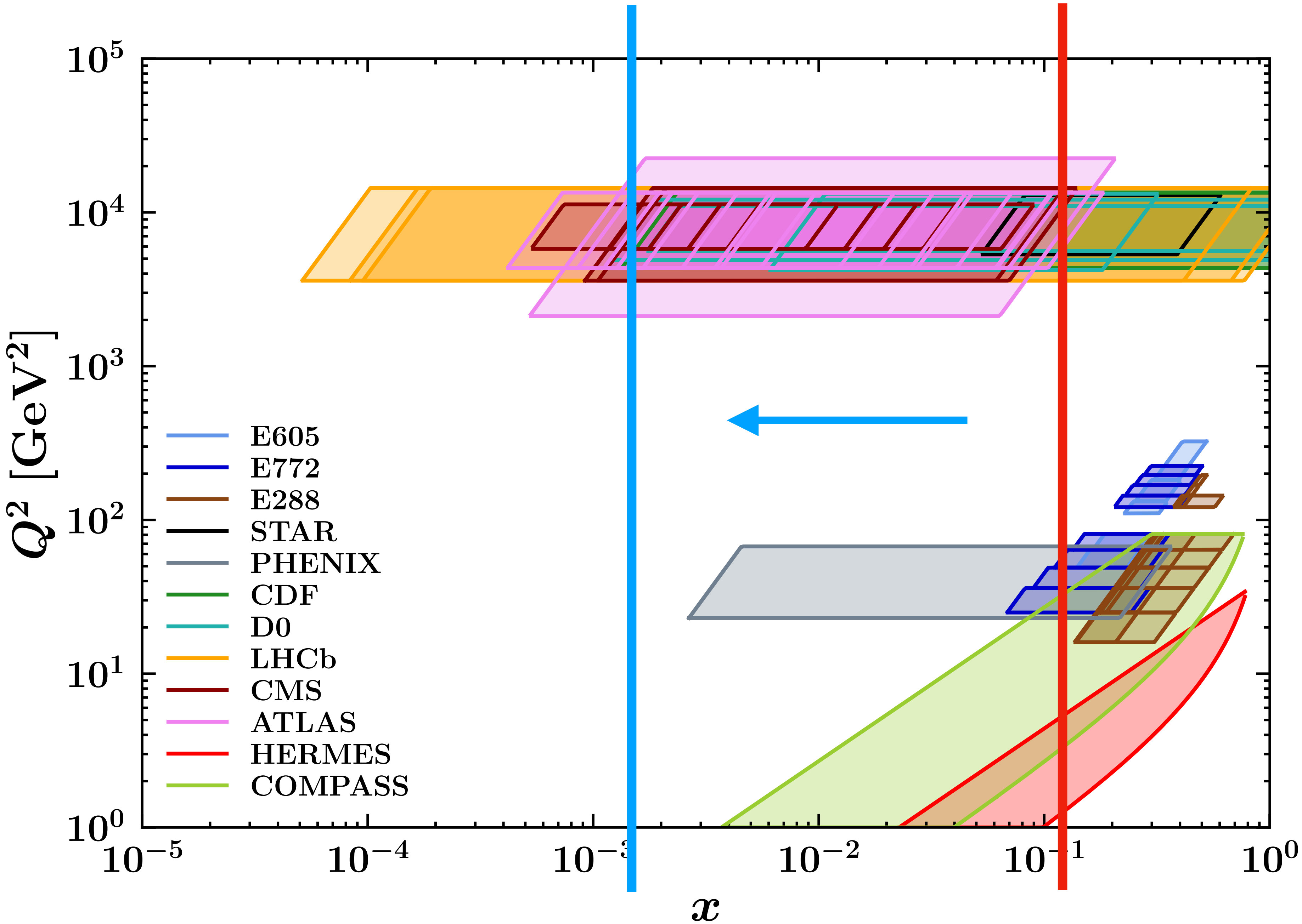


Example I: fit E288 (only) vs fit E605 (only)



Must extrapolate to smaller values of x

Example I: comparing postdictions



Plot from (MAP collaboration):
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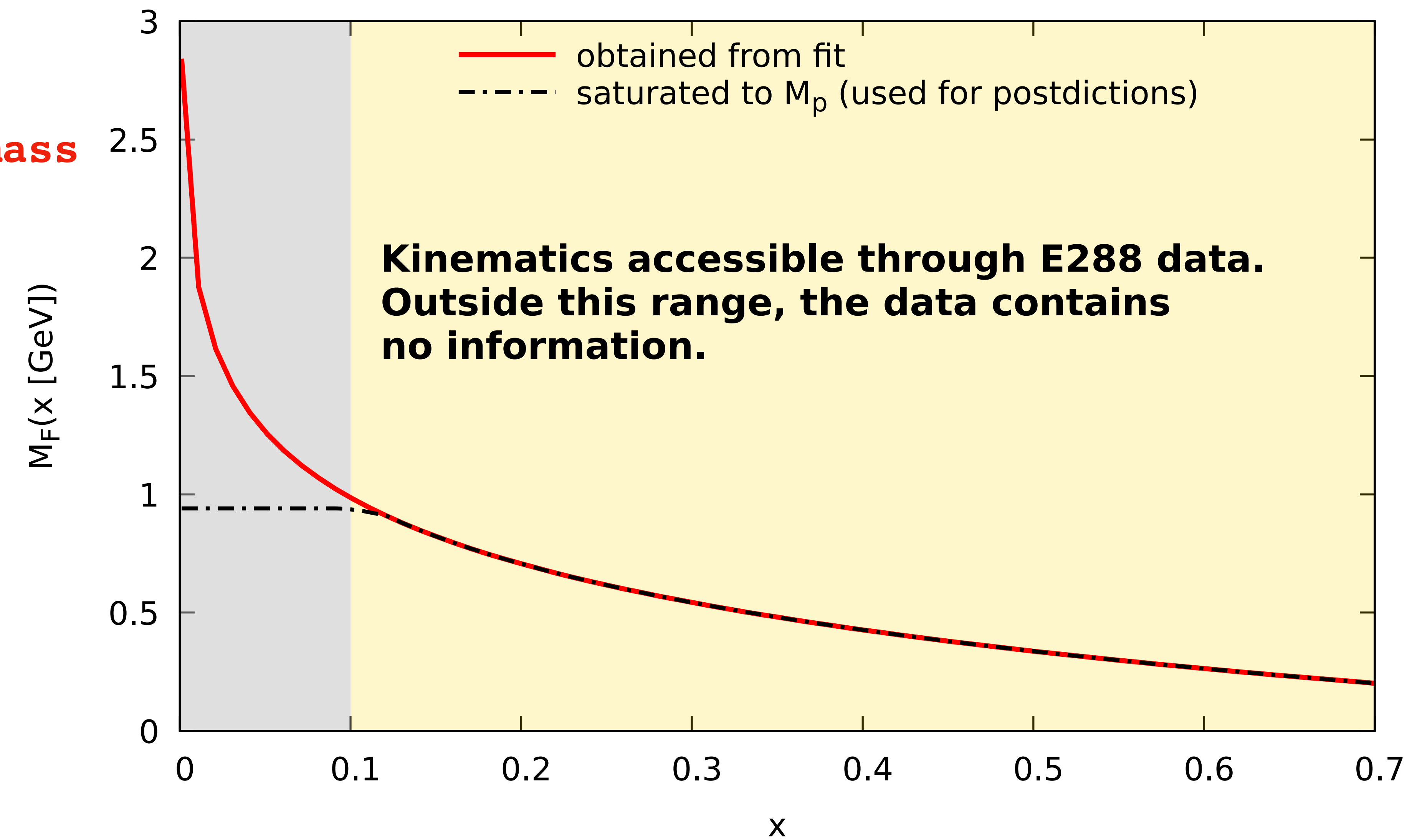
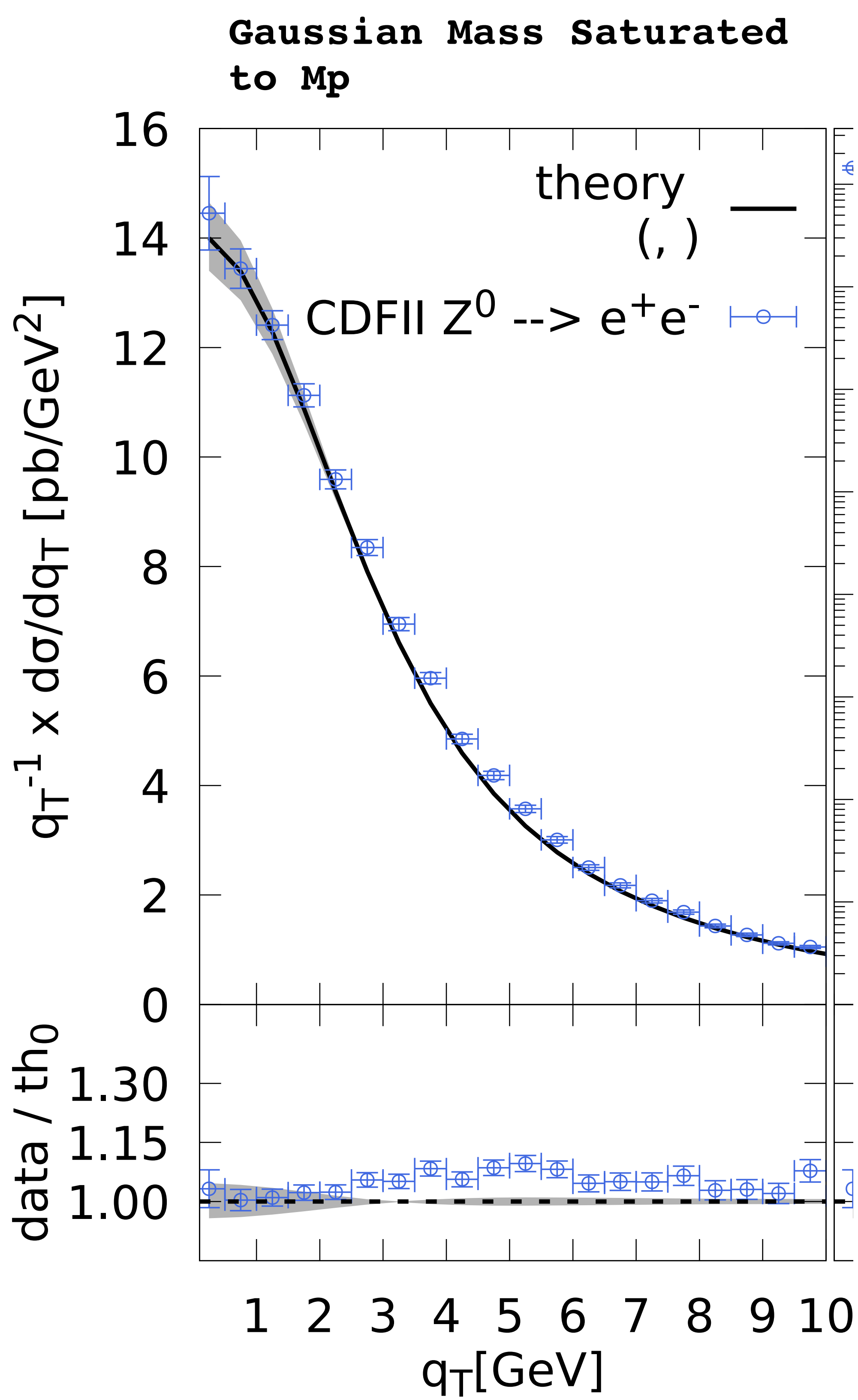
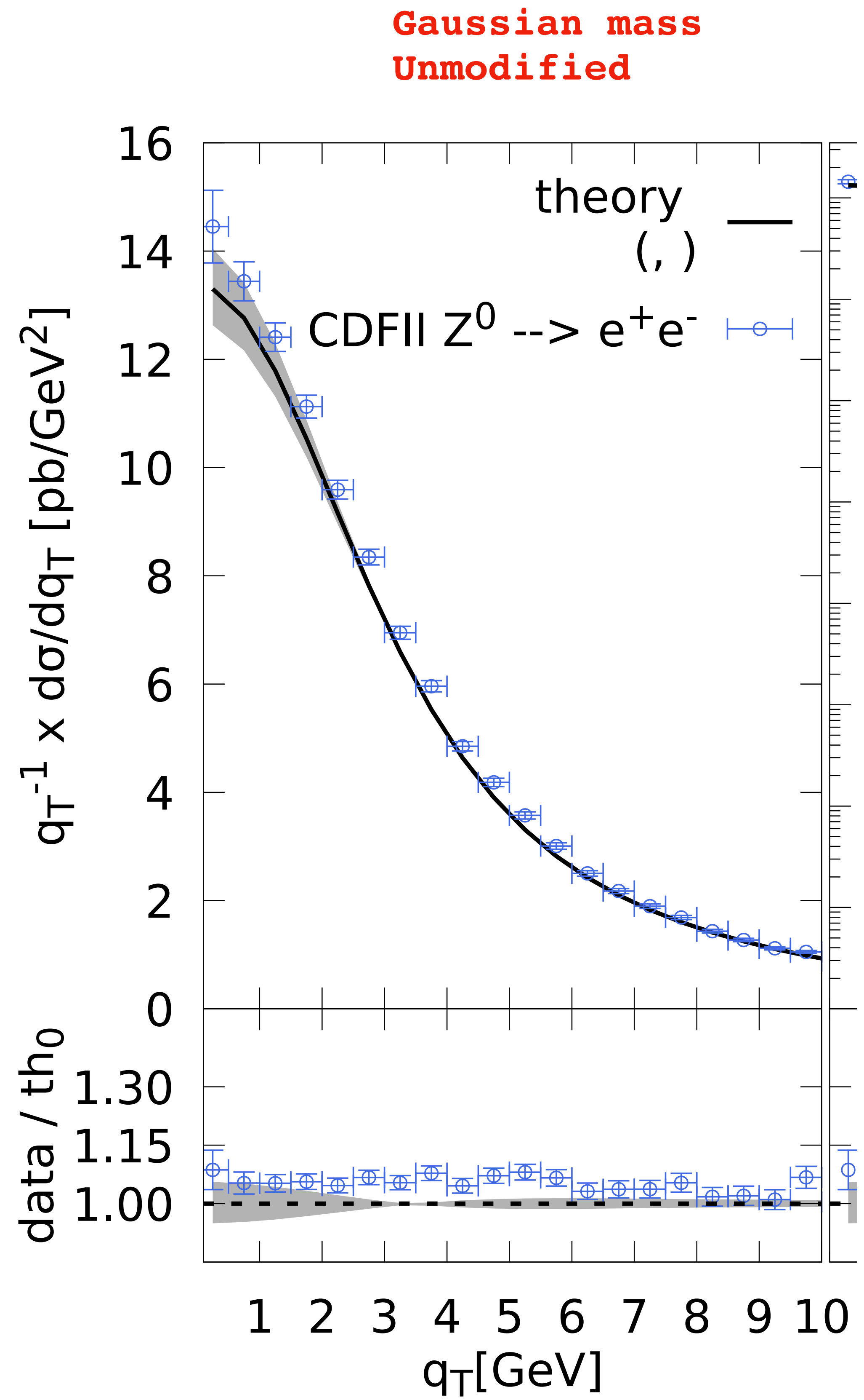
Gaussian mass to large

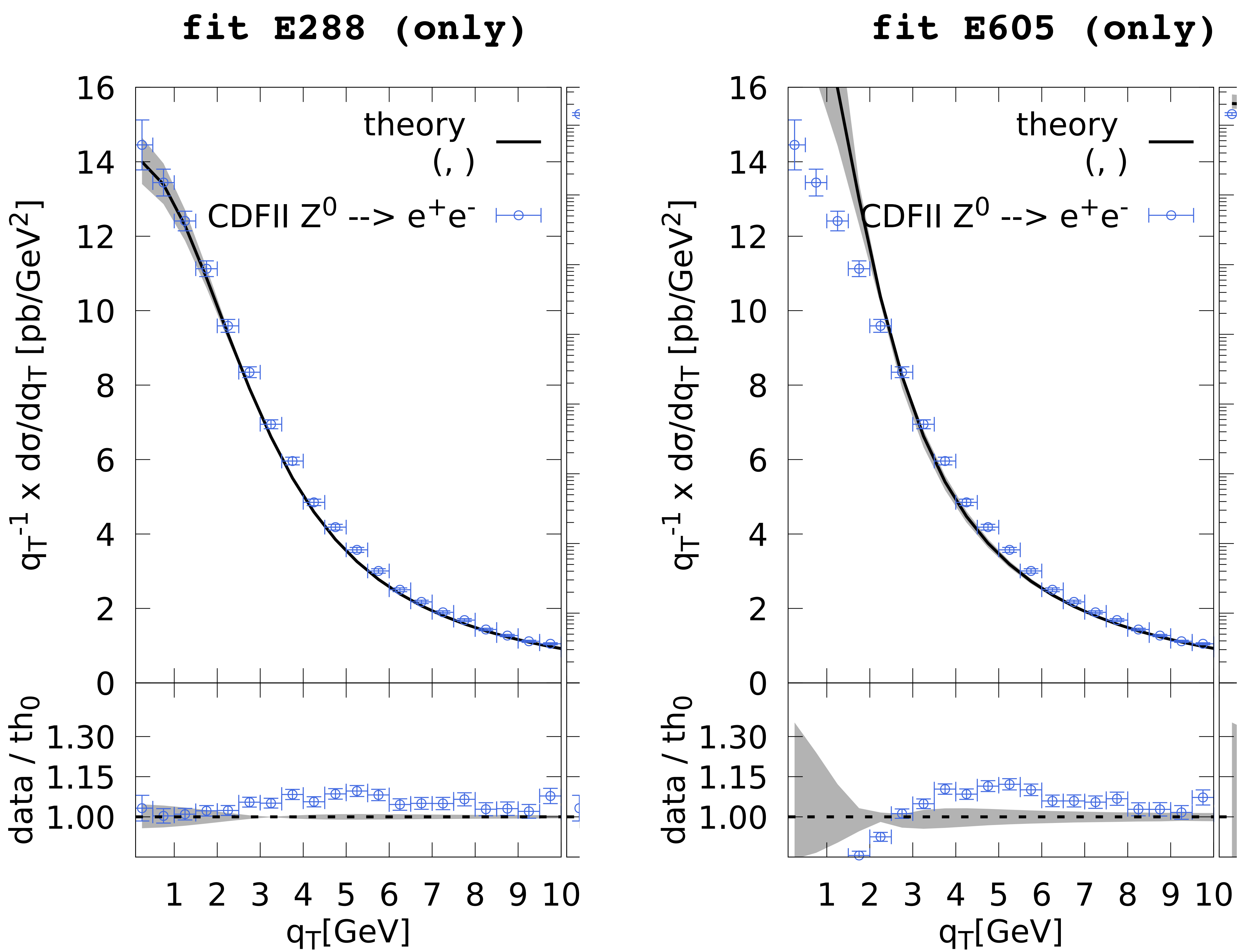


Saturate to keep consistency with pQCD tail

fit E288 (only)



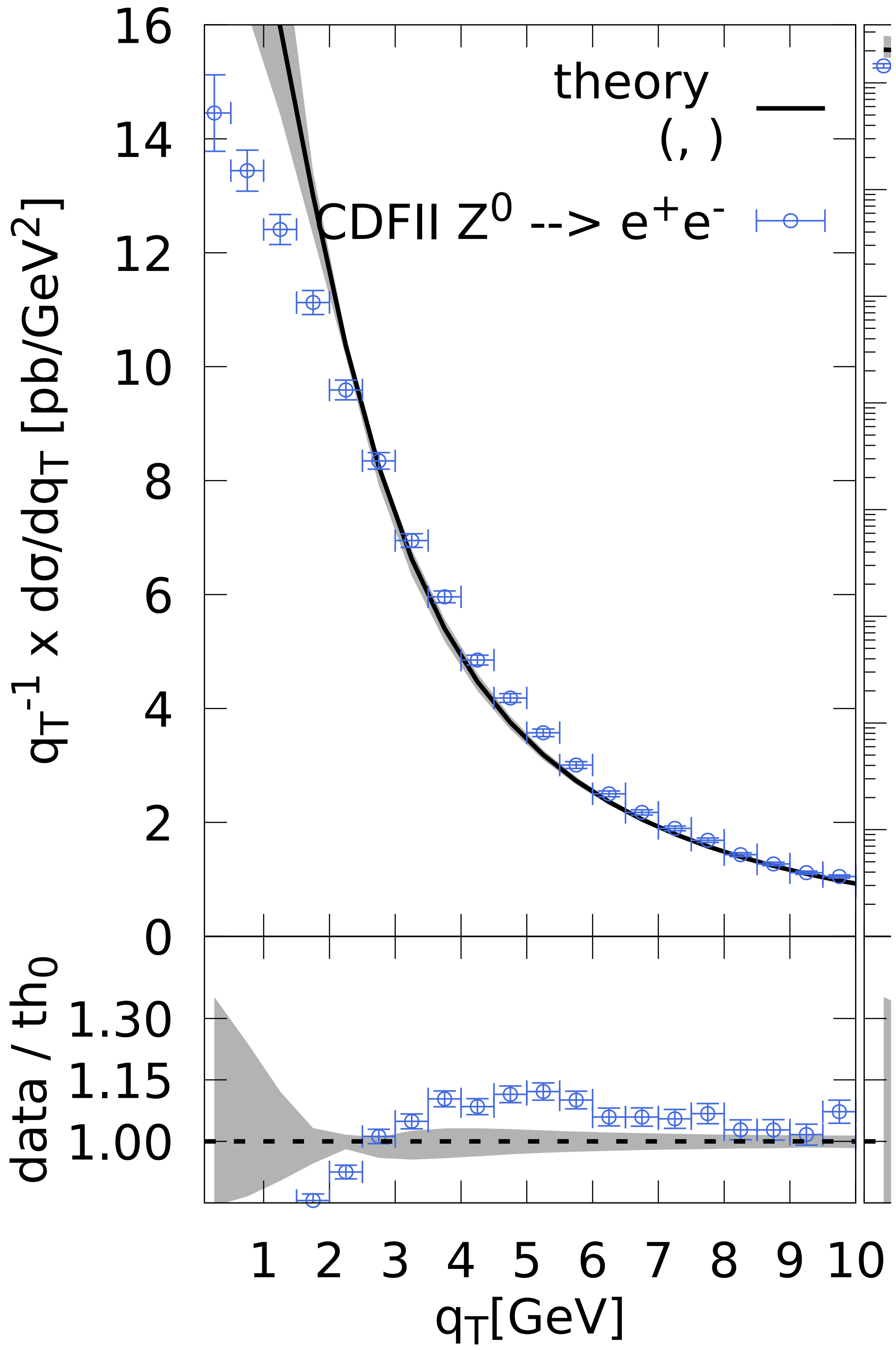




fit E288 (only)
fit E605 (only)
theory
(,)
CDFII Z0 --> e+e-
q_T^-1 x dσ/dq_T [pb/GeV2]
data / th0
q_T[GeV]

**E605 only one energy beam.
Not likely to determine
both TMD and kernel from
this set alone.**

fit E605 (only)



Example II: fit E288 (only). Gaussian vs spectator

Models for TMD core functions (same kernel as before)

$$f^{\text{Gauss}}_{\text{core},i/p}(x, \boldsymbol{k}_\text{T}; Q_0^2) = \frac{e^{-k_\text{T}^2/M_\text{F}^2}}{\pi M_\text{F}^2} \qquad M_\text{F} \to M_0 + M_1 \log(1/x)\,,$$

Free parameters M_0, M_1 , b_k

Same number of parameters

$$f^{\text{Spect}}_{\text{core},i/p}(x, \boldsymbol{k}_\text{T}; Q_0^2) = \frac{1}{\pi} \frac{6\,L^6}{L^2 + 2(m_q + x\,M_p)^2} \frac{k_\text{T}^2 + (m_q + x\,M_p)^2}{(k_\text{T}^2 + L^2)^4}$$

$$L^2 = (1-x)\Lambda^2 + x M_X^2 - x(1-x)M_p^2$$

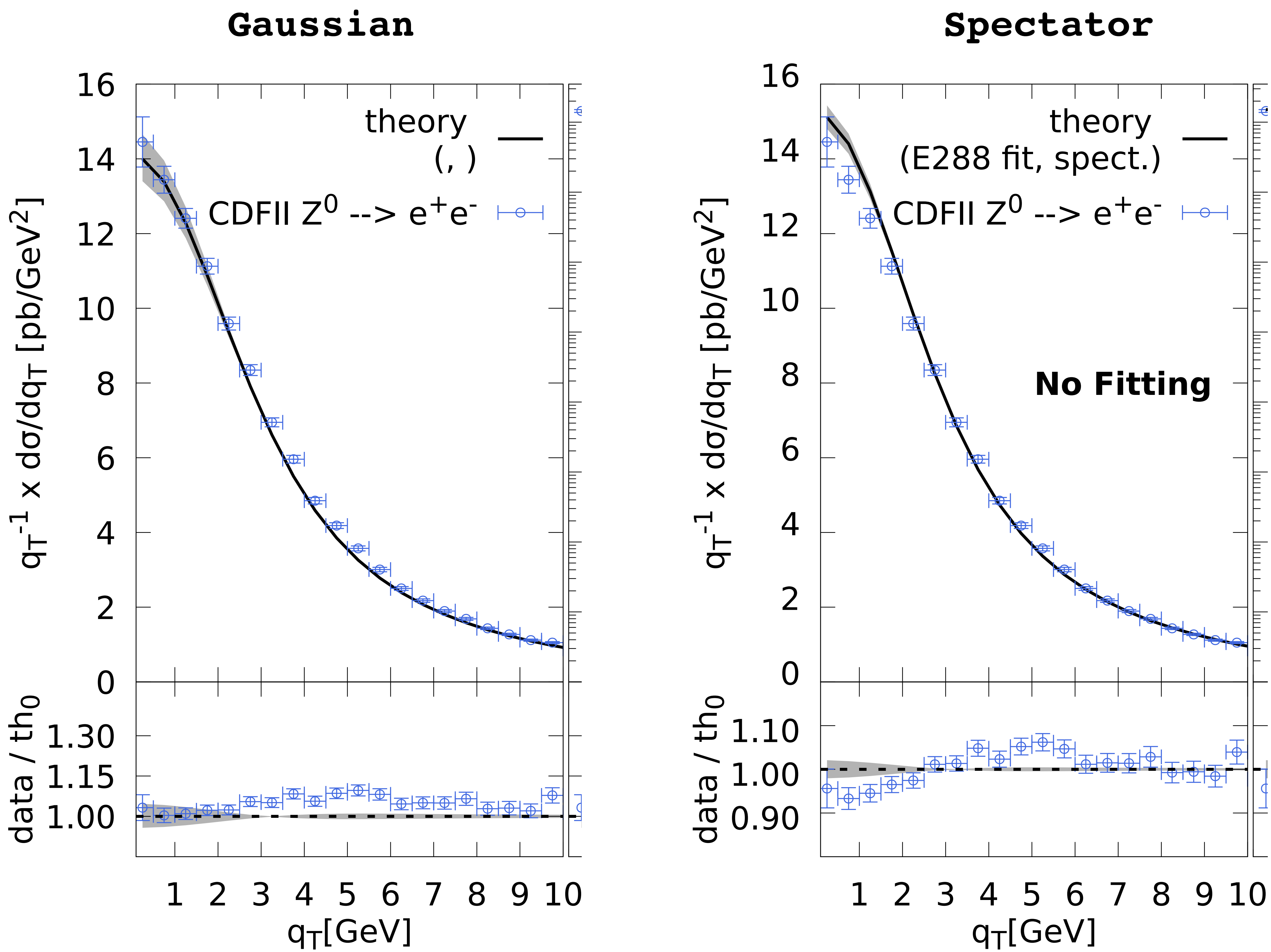
Free parameters Λ , M_X , b_k $\qquad$ **m_q = 0**

Gaussian	
	E288 (130 pts.)
$\chi^2_{\rm dof}$	1.04
M_0 (GeV)	0.0576
M_1 (GeV)	0.403
b_K	2.12
N(nuisance)	1.29

Spectator model fit	
	E288 (130 pts.)
$\chi^2_{\rm dof}$	1.04
Λ (GeV)	0.801
M_X (GeV)	0.438
b_K	1.90
N(nuisance)	1.23

Same χ^2/dof

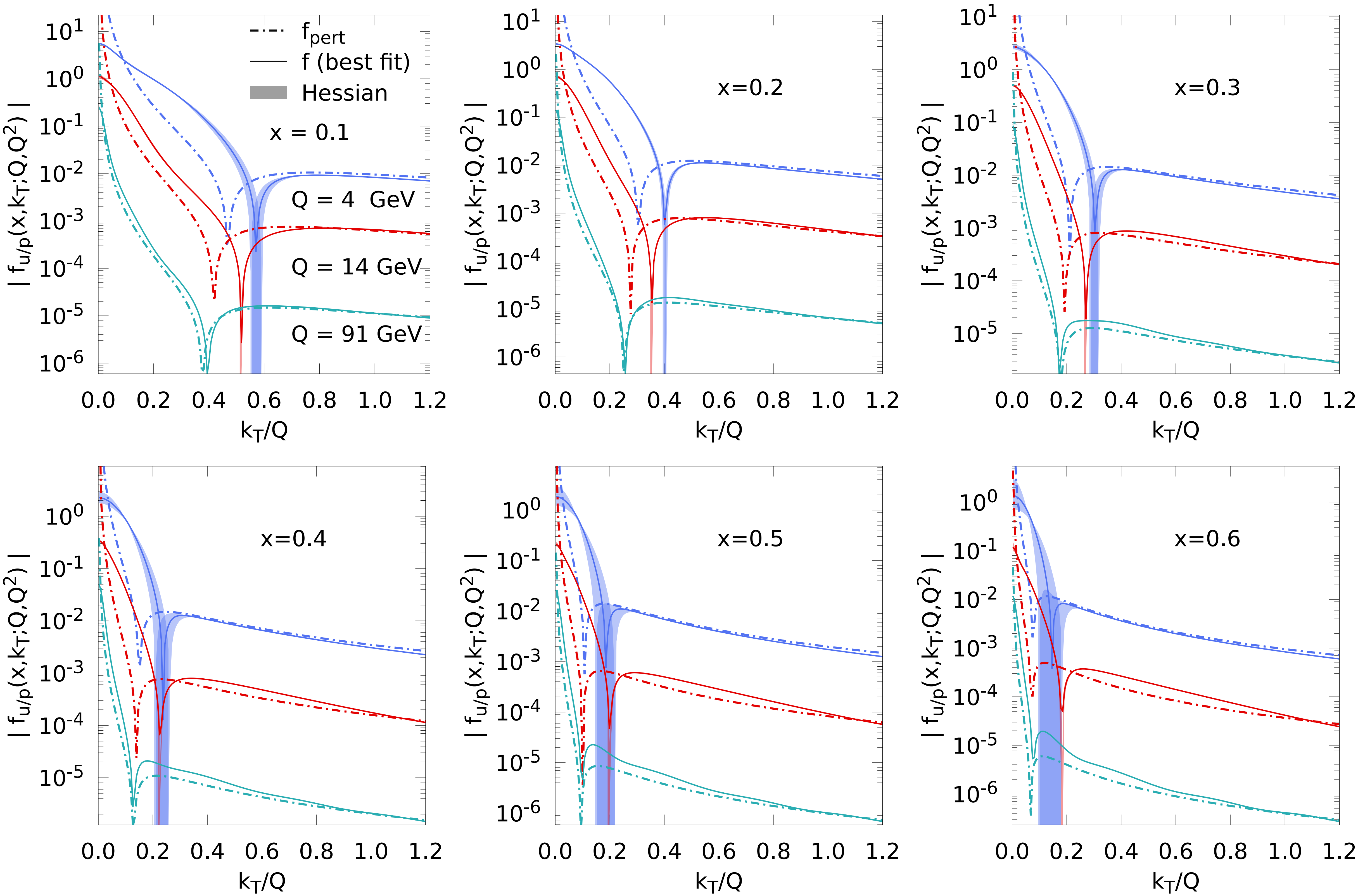
Same no. of parameters



Gaussian
Spectator
theory
(,)
CDFII Z0 --> e+e-
theory
(E288 fit, spect.)
CDFII Z0 --> e+e-
No Fitting
qT-1 x dσ/dqT [pb/GeV2]
data / th0
qT[GeV]

Examples here somewhat qualitative

Examples TMDs Gaussian fit to E288



HSO Strategy (and final remarks)

-Use theoretical constraints, don’t trust the fit will do this job by itself.

-Check/improve constraints

-Prioritize the role of lower scale data (more information about intrinsic kT)

-Emphasize the predictive aspect of factorization theorems

Thanks

$$[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}} \boxed{\tilde{f}_{j/p}(x,\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0},\mu_{Q_0}^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_{\mathrm{T}};\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0},\mu_{Q_0}^2)}$$
$$\times \exp\left\{2\int_{\mu_{Q_0}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{Q_0^2}\tilde{K}(\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0})\right\}.$$

$$[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_{\mathrm{T}}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_{\mathrm{T}}\cdot\boldsymbol{b}_{\mathrm{T}}}\ \tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}^2)\ \tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{b_*},\mu_{b_*}^2)$$
$$\times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q}\frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*;\mu_{b_*})\right\}$$
$$\times \exp\left\{-g_{j/p}(x,b_{\mathrm{T}}) - g_{h/j}(z,b_{\mathrm{T}}) - g_K(b_{\mathrm{T}})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}.$$

Same formula, just reorganized

$$-g_{j/p}(x,b_{\mathrm{T}}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x,\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0},Q_0^2)}{\tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right), \qquad -g_{h/j}(z,b_{\mathrm{T}}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z,\boldsymbol{b}_{\mathrm{T}};\mu_{Q_0},Q_0^2)}{\tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right),$$

$$g_K(b_{\mathrm{T}}) \equiv \tilde{K}(b_*;\mu) - \tilde{K}(b_{\mathrm{T}};\mu)\,.$$

Precise definitions for g functions, $b_*(b_T)$ is a transition function bounded by some b_{max}. Note that b_* dependence cancels exactly. It is really unimportant which b_* we choose.

$$
\begin{aligned}
[f_{j/p}, D_{h/j}] \to & \int \frac{\mathrm{d}^2 \boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\, \tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2)\, \tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{b_*}, \mu_{b_*}^2) \\
& \times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu')) \right] + \ln\frac{Q^2}{\mu_{b_*}^2}\tilde{K}(b_*; \mu_{b_*}) \right\} \\
& \times \exp\left\{ -g_{j/p}(x, b_\mathrm{T}) - g_{h/j}(z, b_\mathrm{T}) - g_K(b_\mathrm{T}) \ln\left(\frac{Q^2}{Q_0^2}\right) \right\} .
\end{aligned}
$$

Same formula, just reorganized

$$
-g_{j/p}(x, b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)} \right) , \qquad -g_{h/j}(z, b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z, \boldsymbol{b}_\mathrm{T}; \mu_{Q_0}, Q_0^2)}{\tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)} \right) ,
$$

$$
g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_\mathrm{T}; \mu) .
$$

$$
\boxed{\boldsymbol{b}_*(b_\mathrm{T}) = \frac{\boldsymbol{b}_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2/b_{\mathrm{max}}^2}} ,}
$$

Precise definitions for g functions, $b_*(b_T)$ is a transition function bounded by some b_{max}. Note that b_* dependence cancels exactly. High sensitivity to b_* or b_{max} signals an issue.

$$\begin{aligned}\left[f_{j/p}, D_{h/j}\right] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; \tilde{f}^{\mathrm{OPE}}_{j/p}(x, \boldsymbol{b}_*; \mu_{b_*}, \mu^2_{b_*})\; \tilde{D}^{\mathrm{OPE}}_{h/j}(z, \boldsymbol{b}_*; \mu_{b_*}, \mu^2_{b_*}) \\ \times \exp\left\{2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu^2_{b_*}}\tilde{K}(b_*; \mu_{b_*})\right\} \\ \times \exp\left\{-g_{j/p}(x, b_\mathrm{T}) - g_{h/j}(z, b_\mathrm{T}) - g_K(b_\mathrm{T})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\} \boxed{+\, O(b_{\max}\, m)}\end{aligned}$$

$\longleftarrow$ errors

Use of OPE introduces errors. Must keep b_{max} reasonably small.

$$\boxed{\frac{\mathrm{d}}{\mathrm{d}b_{\max}}\left[f_{j/p}, D_{h/j}\right] = O\left(m b_{\max}\right)}$$

$$[f_{j/p}, D_{h/j}] \to \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2}\, e^{-i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}}\; \tilde{f}^{\mathrm{OPE}}_{j/p}(x,\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*})\; \tilde{D}^{\mathrm{OPE}}_{h/j}(z,\boldsymbol{b}_*;\mu_{b_*},\mu^2_{b_*})$$

$$\times \exp\left\{ 2\int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\frac{Q}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q^2}{\mu^2_{b_*}}\tilde{K}(b_*;\mu_{b_*})\right\}$$

Models $\to$ $$\boxed{\times \exp\left\{-g_{j/p}(x,b_\mathrm{T}) - g_{h/j}(z,b_\mathrm{T}) - g_K(b_\mathrm{T})\ln\left(\frac{Q^2}{Q_0^2}\right)\right\}} + O(b_{\max}\, m)$$

Definitions:
Smooth transition
to small-b_T region
by construction

$$-g_{h/j}(z,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{D}_{h/j}(z,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)$$

$$-g_{j/p}(x,b_\mathrm{T}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x,\boldsymbol{b}_\mathrm{T};\mu_{Q_0},Q_0^2)}{\tilde{f}_{j/p}(x,\boldsymbol{b}_*;\mu_{Q_0},Q_0^2)}\right)$$

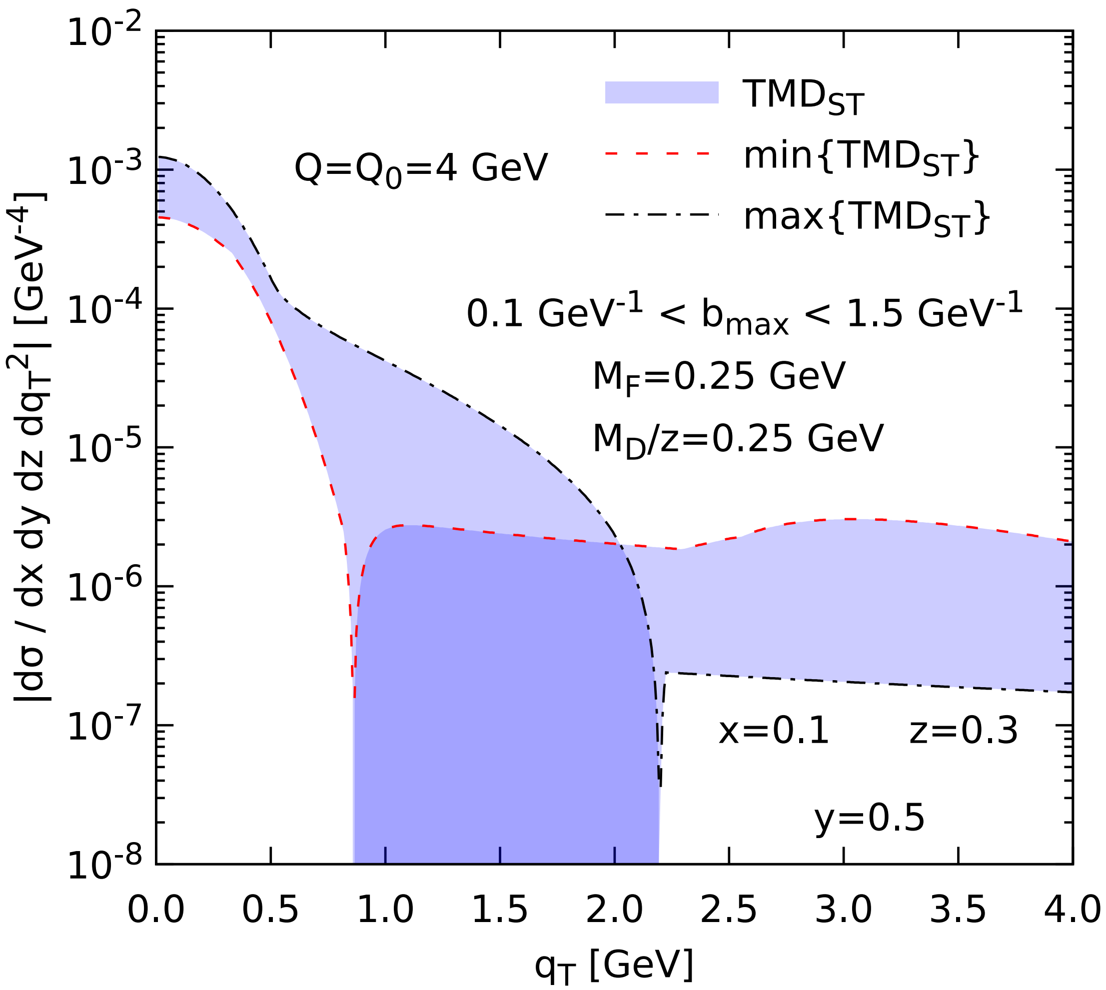
$$g_K(b_\mathrm{T}) \equiv \tilde{K}(b_*;\mu) - \tilde{K}(b_\mathrm{T};\mu)\,.$$

Typical choices:
generally unconstrained

$$g_{h/j}(z,b_\mathrm{T}) = \frac{1}{4\,z^2} M_D^2\, b_\mathrm{T}^2$$

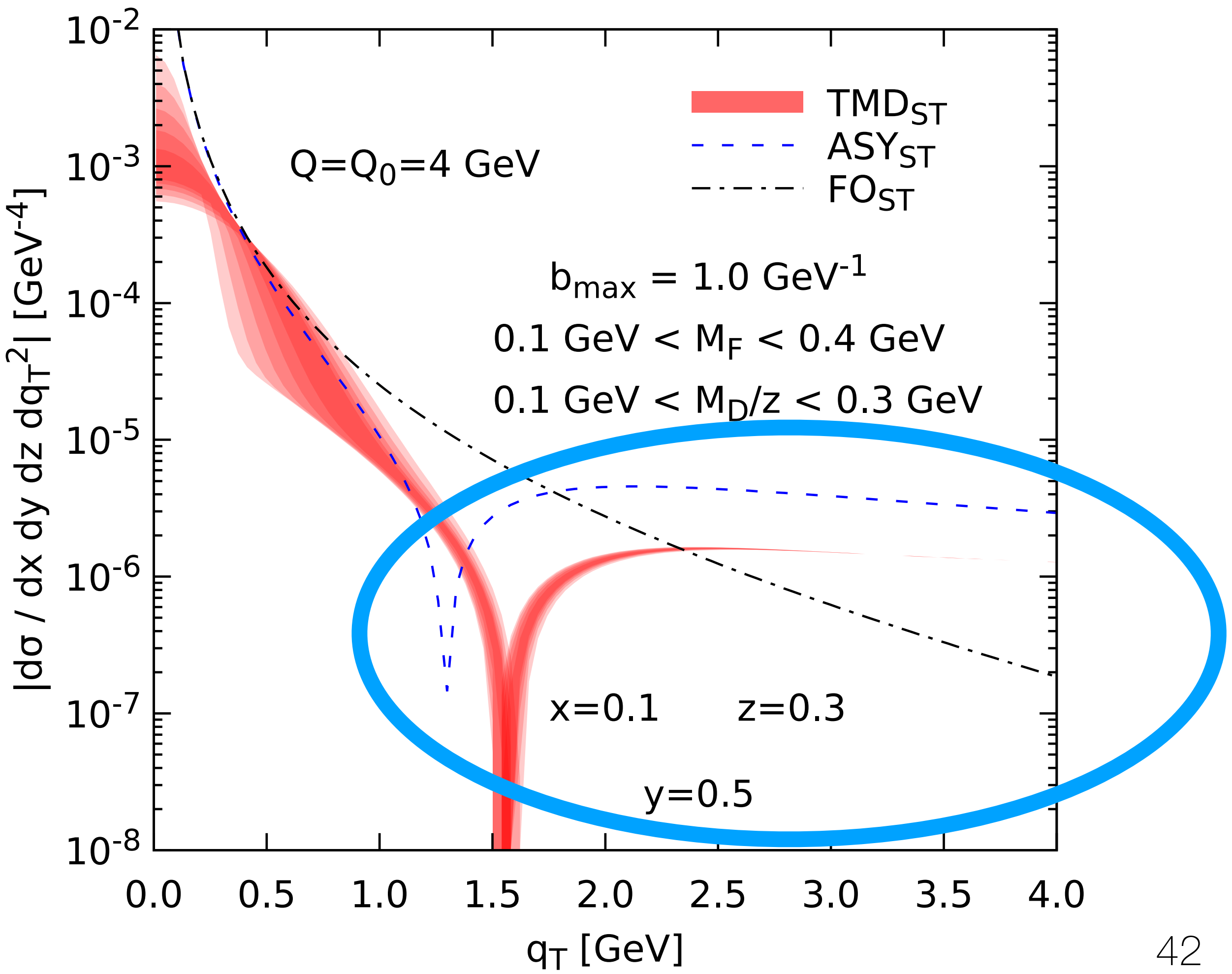
$$g_{j/p}(x,b_\mathrm{T}) = \frac{1}{4}\, M_F^2\, b_\mathrm{T}^2$$

$$g_K(b_\mathrm{T}) = \frac{g_2}{2\,M_K^2}\ln\left(1 + M_K^2 b_\mathrm{T}^2\right)$$



Issues:

Note the large-q_T (small-b_T) region should be determined by the OPE. **Small mass parameters can't really compensate for this b_{max} dependence.**

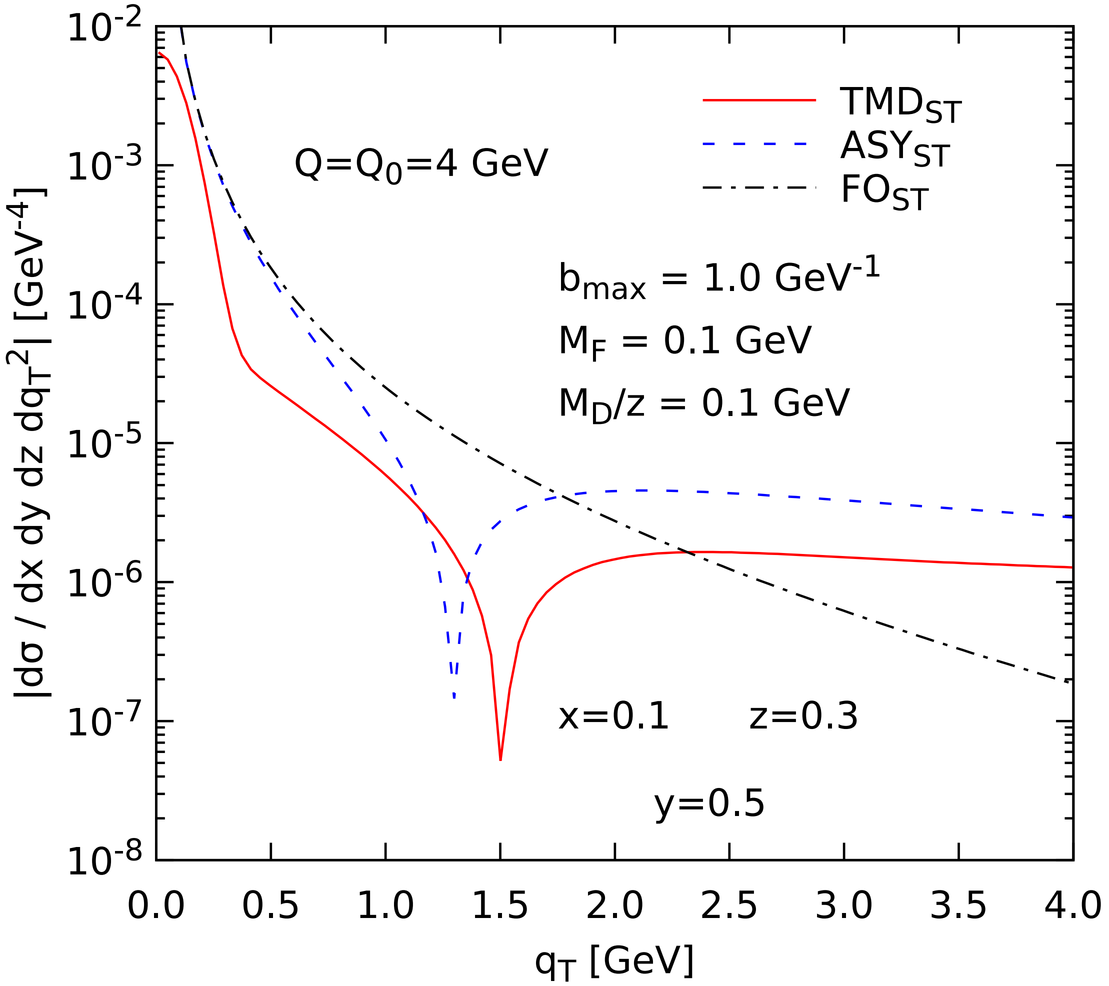


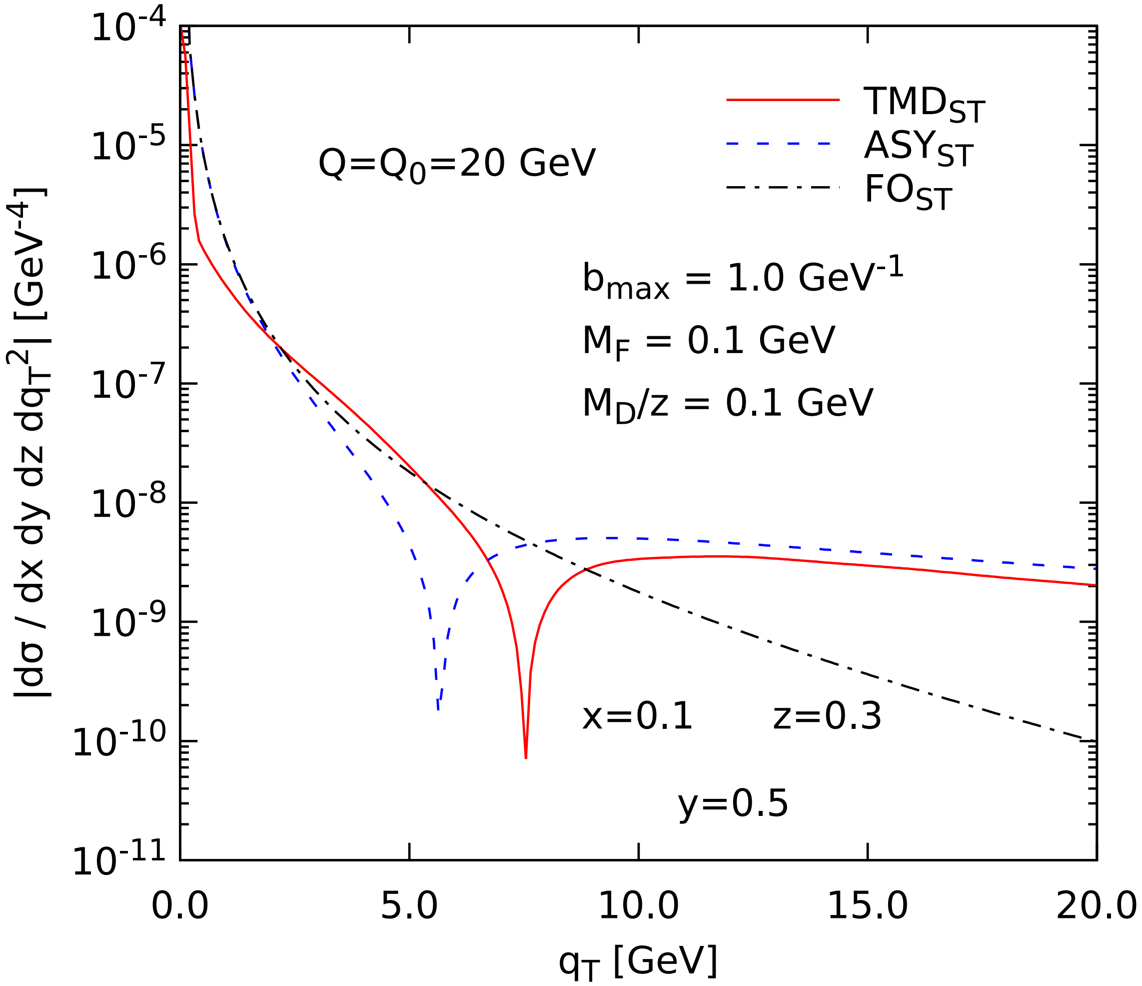
Typical choices:
generally unconstrained

$$g_{h/j}(z, b_{\rm T}) = \frac{1}{4\,z^2} M_D^2\, b_{\rm T}^2$$

$$g_{j/p}(x, b_{\rm T}) = \frac{1}{4}\, M_F^2\, b_{\rm T}^2$$

$$g_K(b_{\rm T}) = \frac{g_2}{2\, M_K^2} \ln\left(1 + M_K^2 b_{\rm T}^2\right)$$





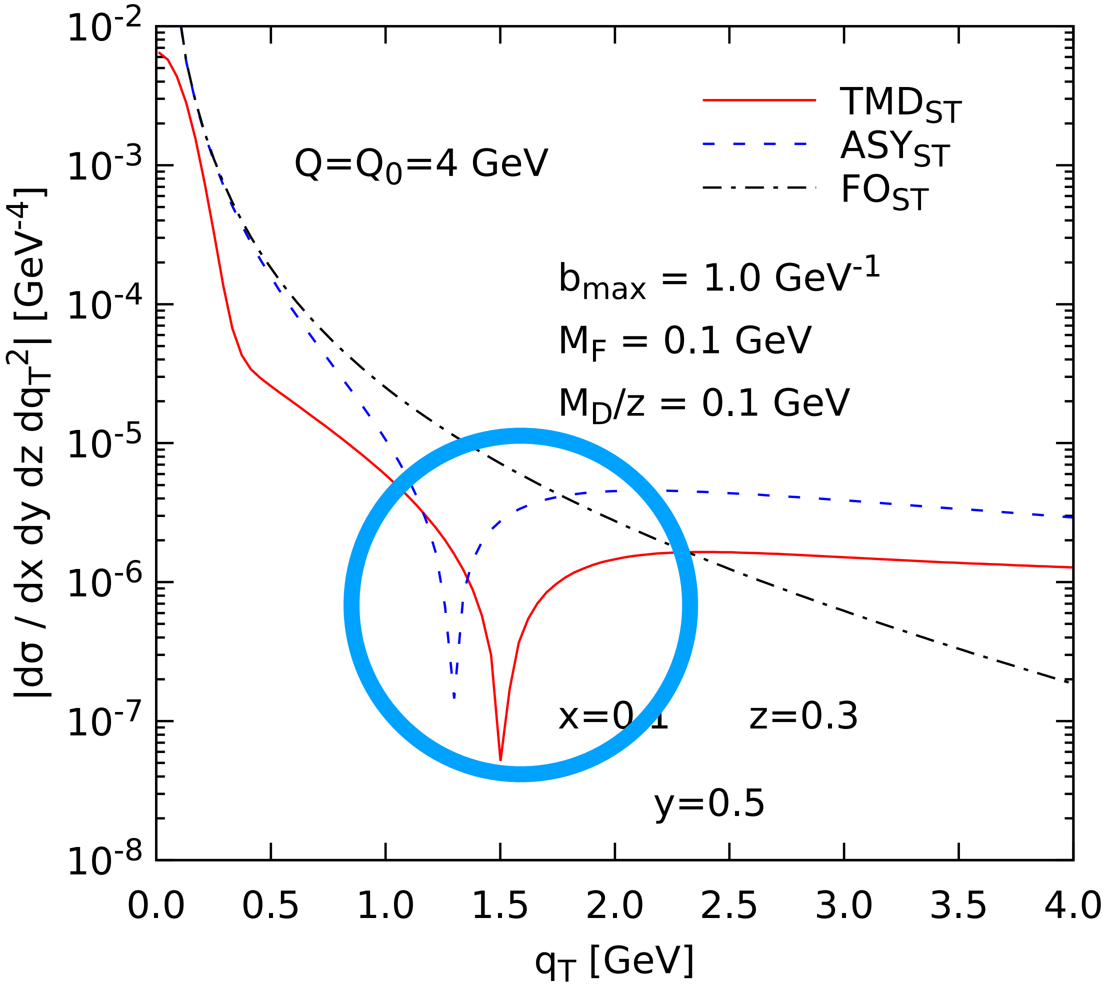
Issues:

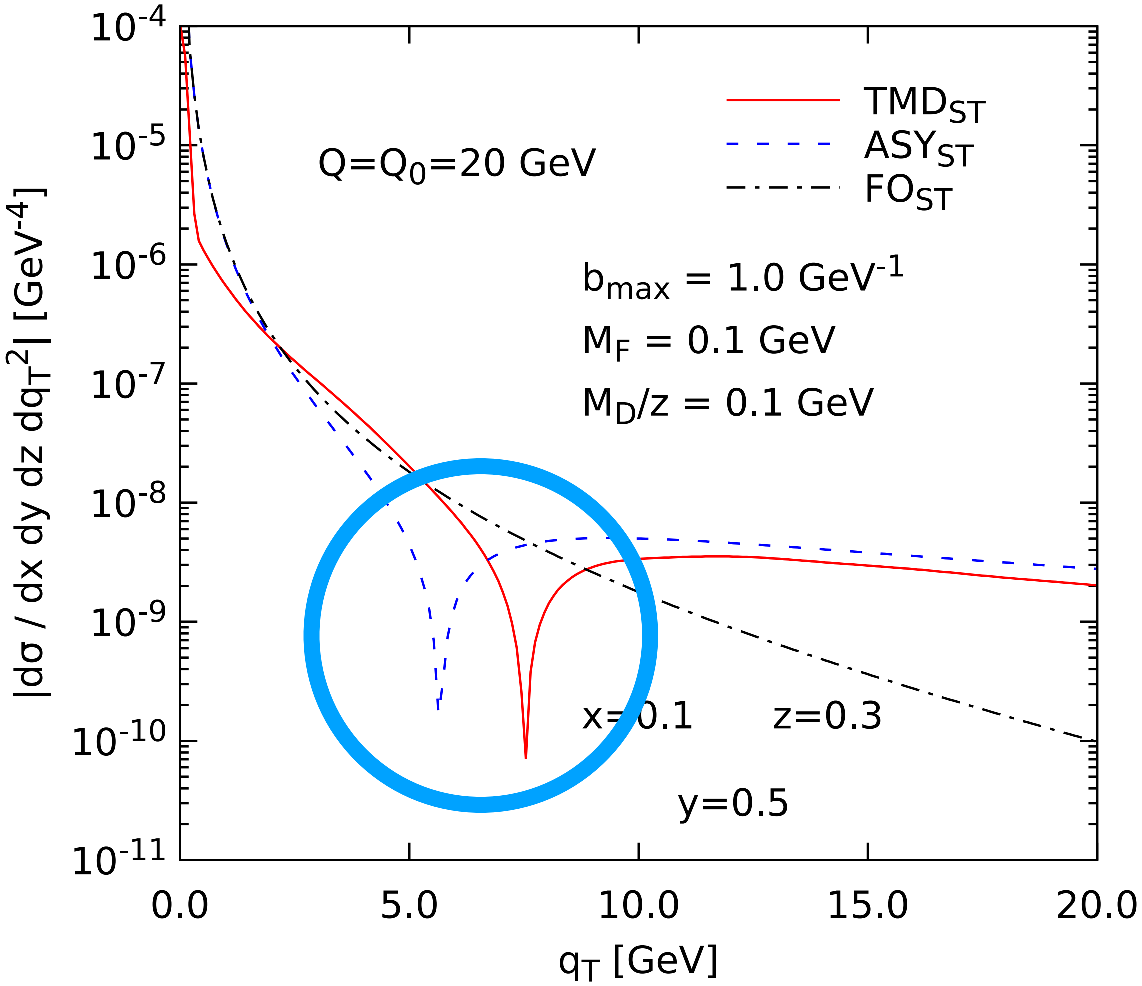
Typical choices:
generally unconstrained

$$g_{h/j}(z, b_{\mathrm{T}}) = \frac{1}{4\,z^2} M_D^2\, b_{\mathrm{T}}^2$$

$$g_{j/p}(x, b_{\mathrm{T}}) = \frac{1}{4}\, M_F^2\, b_{\mathrm{T}}^2$$

$$g_K(b_{\mathrm{T}}) = \frac{g_2}{2\, M_K^2} \ln\left(1 + M_K^2 b_{\mathrm{T}}^2\right)$$





Issues:

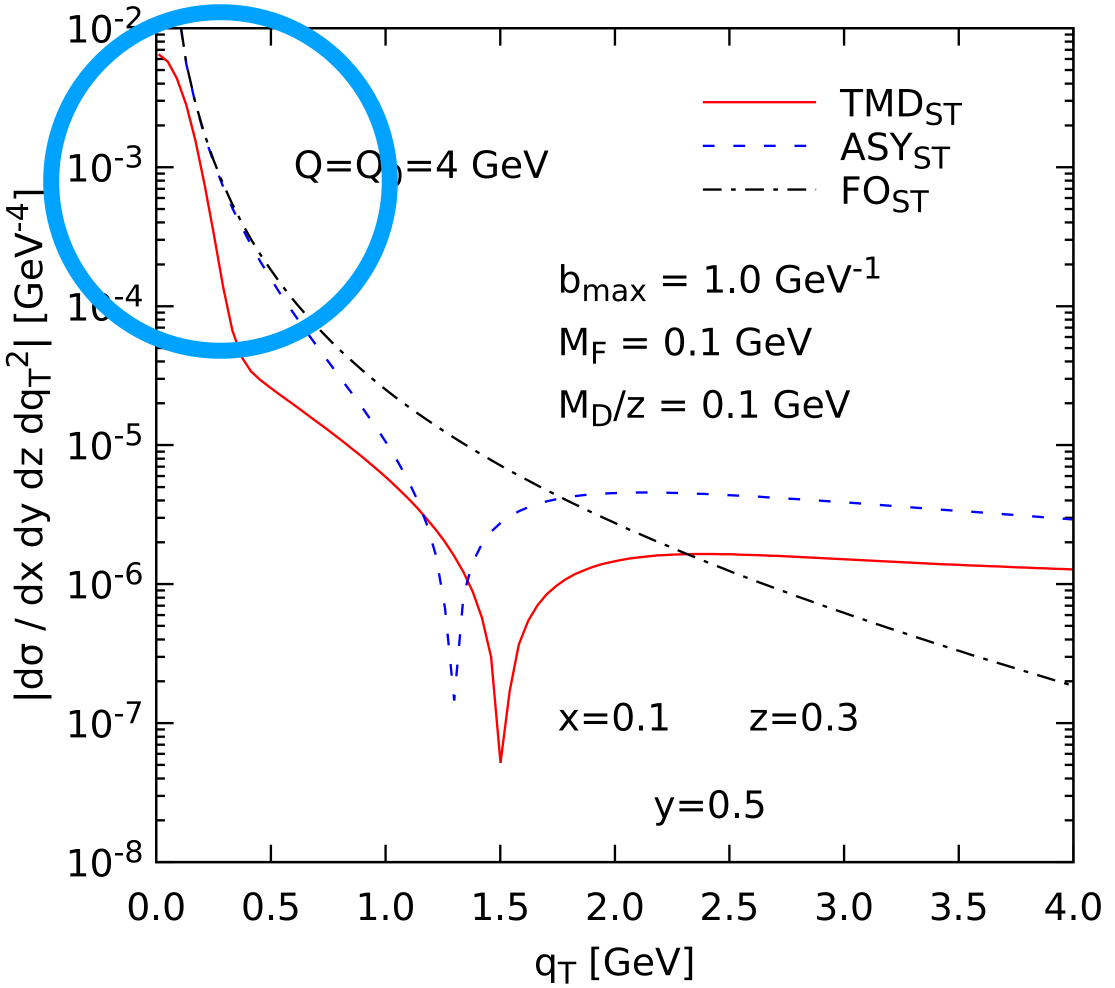
Asymptotic term does not approximate well the TMD term, even at a scale of **Q_0=20 GeV**

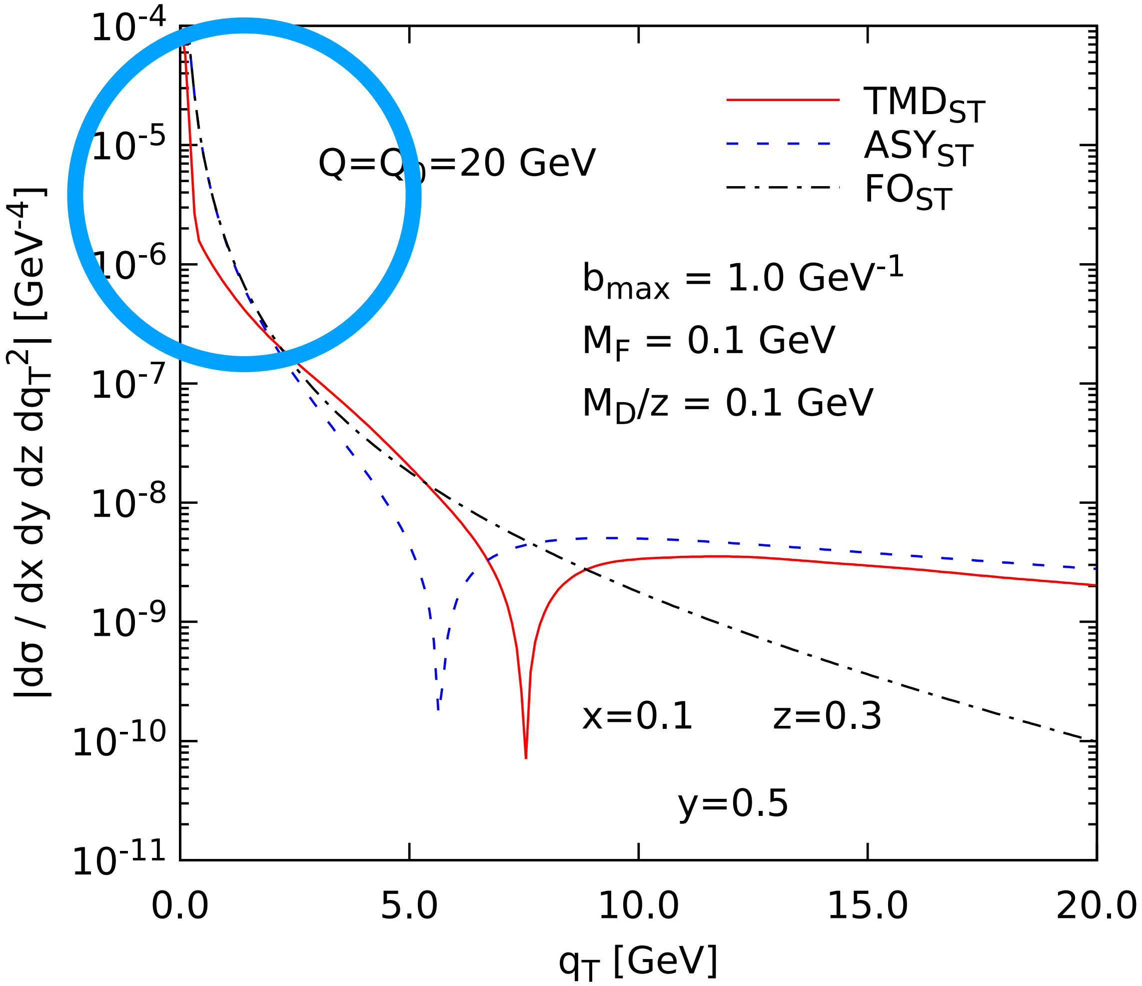
Typical choices:
generally unconstrained

$$g_{h/j}(z, b_\mathrm{T}) = \frac{1}{4\,z^2} M_D^2\, b_\mathrm{T}^2$$

$$g_{j/p}(x, b_\mathrm{T}) = \frac{1}{4}\, M_F^2\, b_\mathrm{T}^2$$

$$g_K(b_\mathrm{T}) = \frac{g_2}{2\,M_K^2} \ln\left(1 + M_K^2 b_\mathrm{T}^2\right)$$





Issues:

No region of "overlap" between TMD term and FO. This means smooth matching is not possible

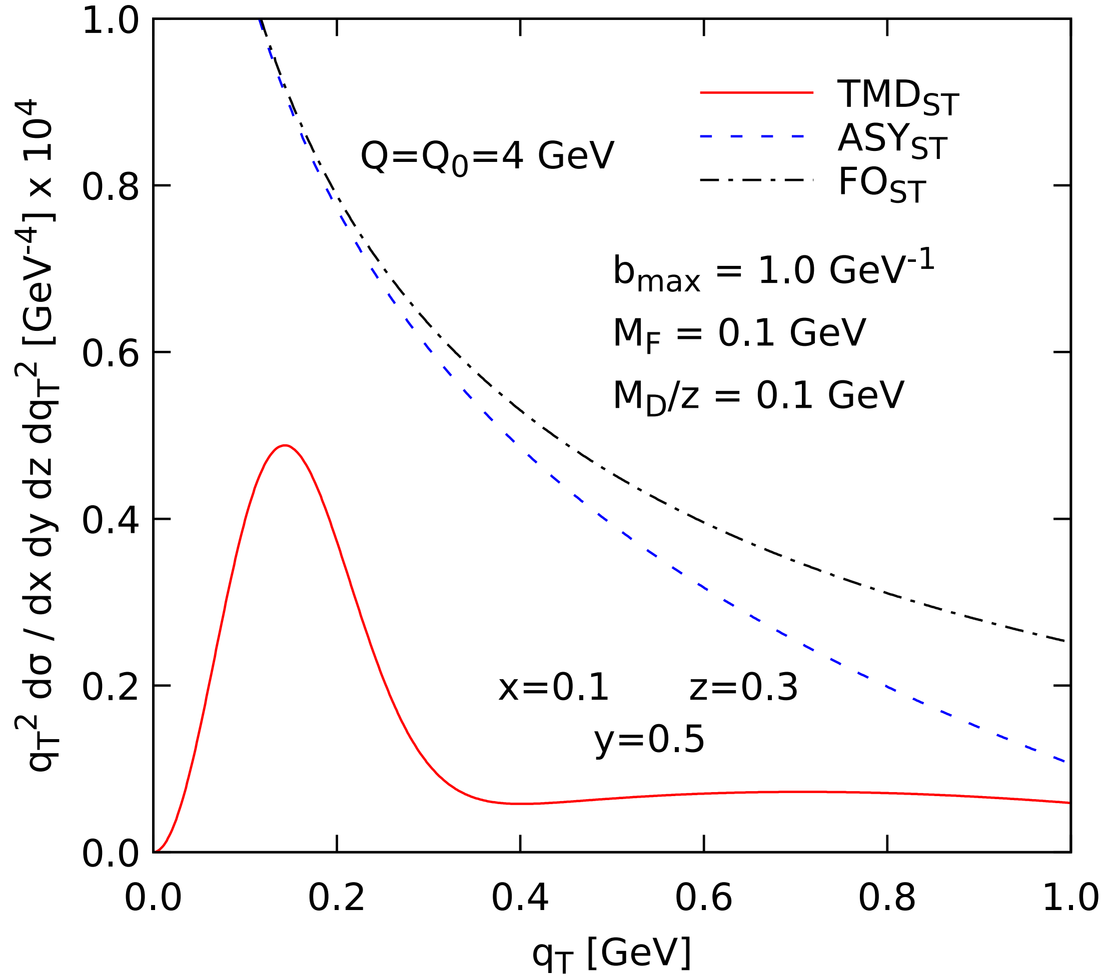
Typical choices:

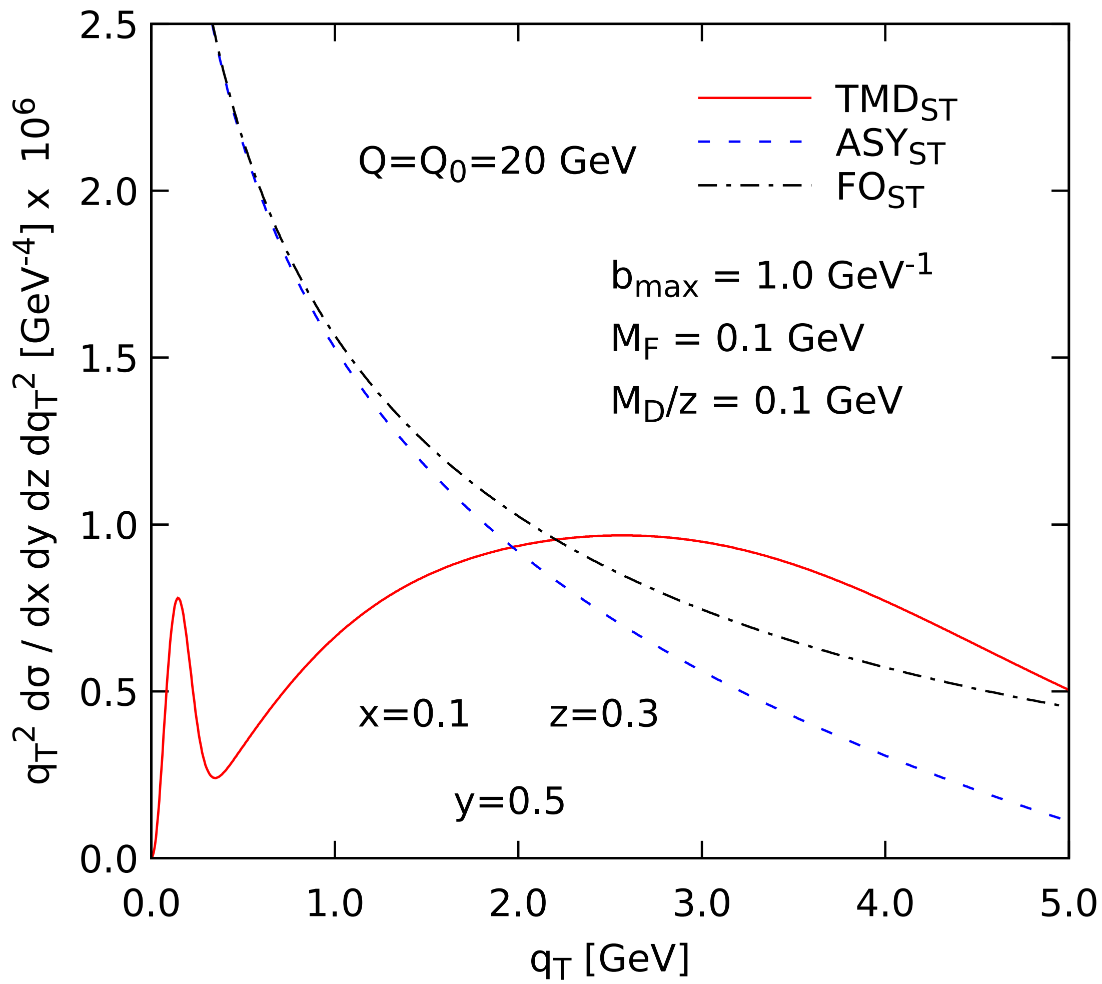
generally unconstrained

$$g_{h/j}(z, b_T) = \frac{1}{4\,z^2} M_D^2\, b_T^2$$

$$g_{j/p}(x, b_T) = \frac{1}{4}\, M_F^2\, b_T^2$$

$$g_K(b_T) = \frac{g_2}{2\,M_K^2} \ln\left(1 + M_K^2 b_T^2\right)$$





Issues:

No region of "overlap" between TMD term and FO. This means smooth matching is not possible

Typical choices:

generally unconstrained

$$g_{h/j}(z, b_{\rm T}) = \frac{1}{4\,z^2} M_D^2\, b_{\rm T}^2$$

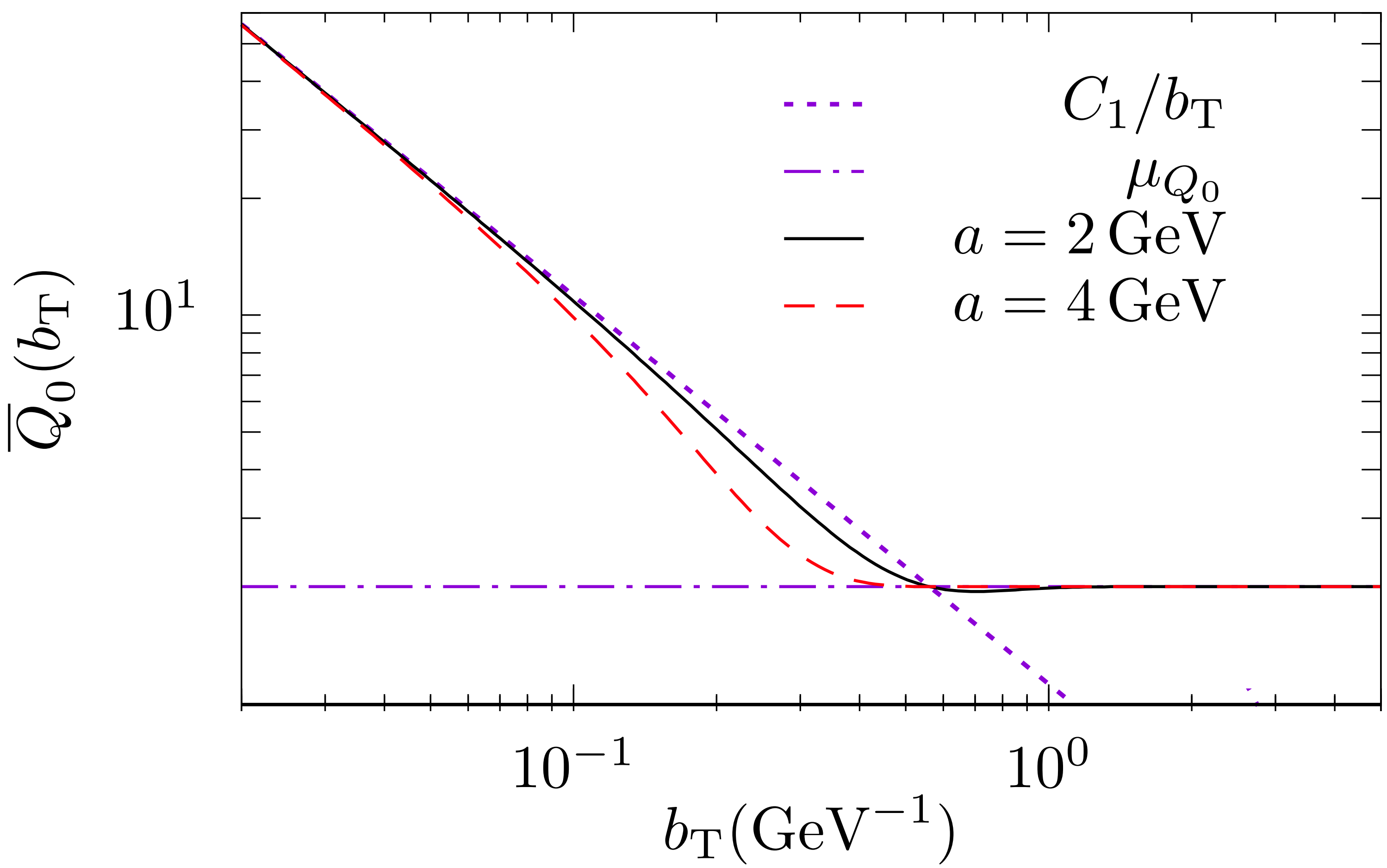
$$g_{j/p}(x, b_{\rm T}) = \frac{1}{4}\, M_F^2\, b_{\rm T}^2$$

$$g_K(b_{\rm T}) = \frac{g_2}{2\,M_K^2} \ln\left(1 + M_K^2 b_{\rm T}^2\right)$$

Scale setting for evolution to large Q

$$\overline{Q}_0(b_T) = Q_0 \text{ GeV} \left[1 - \left(1 - \frac{C_1}{Q_0 b_T} \right) e^{-a^2 b_T^2} \right]$$

$Q_0 = 2\,\text{GeV}$



- ✳ goes as $1/b_T$ for small b_T
- ✳ approaches input scale Q_0 at large b_T
- ✳ analogous to b_* in usual treatment

Model in the HSO approach

Need RG improvements for pheno at Q >> Q_0

$$\sim \alpha_s(Q_0)^n \ln^m \left(\frac{q_{\mathrm{T}}}{Q_0}\right)$$

Wider range of qT available upon evolution to large Q

$$\tilde{f}_{i/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)$$
$$= \tilde{f}_{\mathrm{inpt},i/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_{\bar{Q}_0}, \bar{Q}_0^2) E(\bar{Q}_0/Q_0, b_{\mathrm{T}})$$

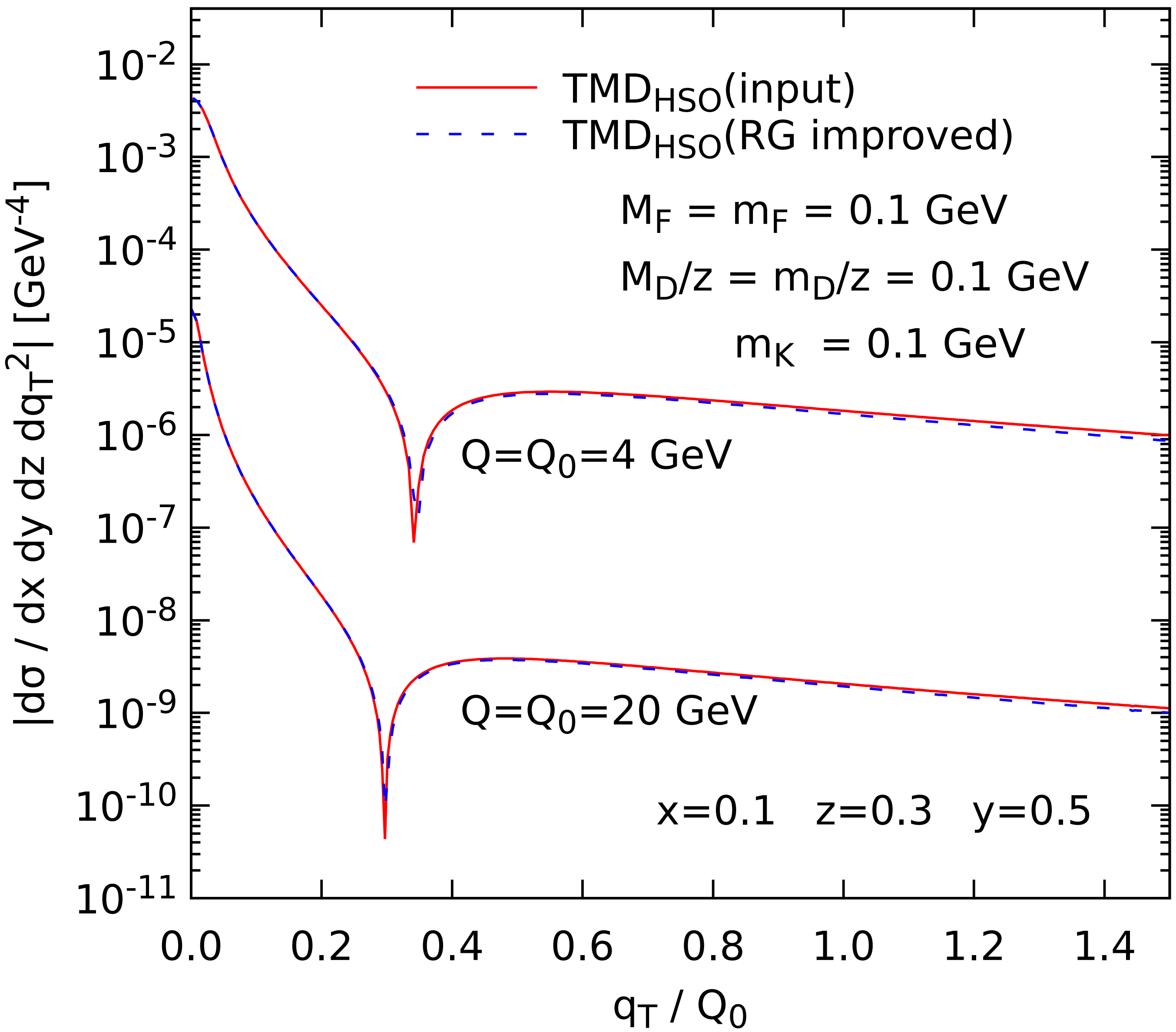
$$\overline{Q}_0(b_{\mathrm{T}}) = Q_0 \text{ GeV} \left[1 - \left(1 - \frac{C_1}{Q_0 b_{\mathrm{T}}}\right) e^{-a^2\, b_{\mathrm{T}}^2}\right]$$

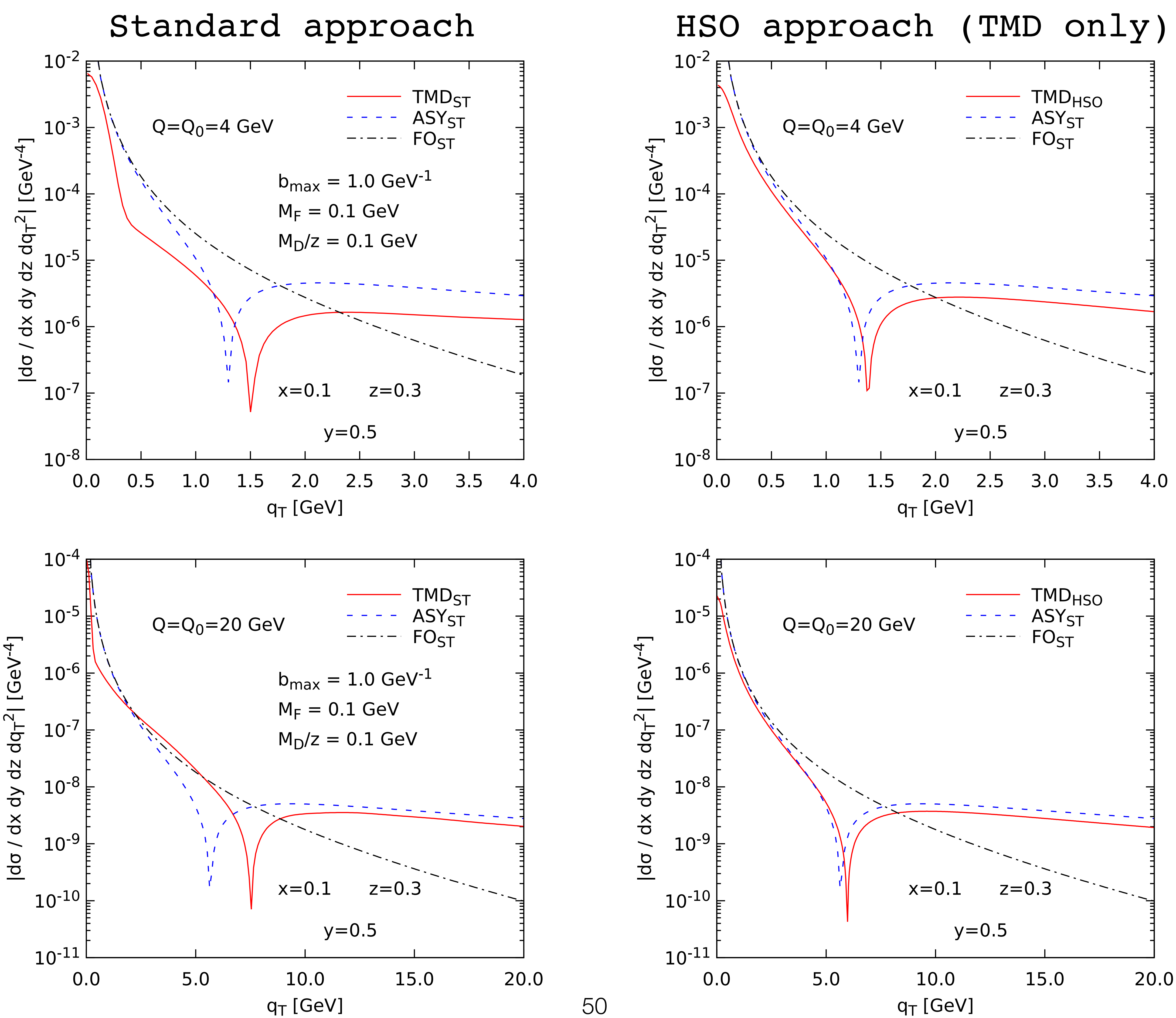
$$E(\bar{Q}_0/Q_0, b_{\mathrm{T}}) \equiv \exp\left\{\int_{\mu_{\bar{Q}_0}}^{\mu_{Q_0}} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \ln\frac{Q_0}{\mu'}\gamma_K(\alpha_s(\mu'))\right] + \ln\frac{Q_0}{\bar{Q}_0}\tilde{K}_{\mathrm{inpt}}(b_{\mathrm{T}}; \mu_{\bar{Q}_0})\right\}.$$

The usual evolution factor

Scale transformation not really needed for pheno at $Q \approx Q_0$

Work with $Q=Q_0$ for now



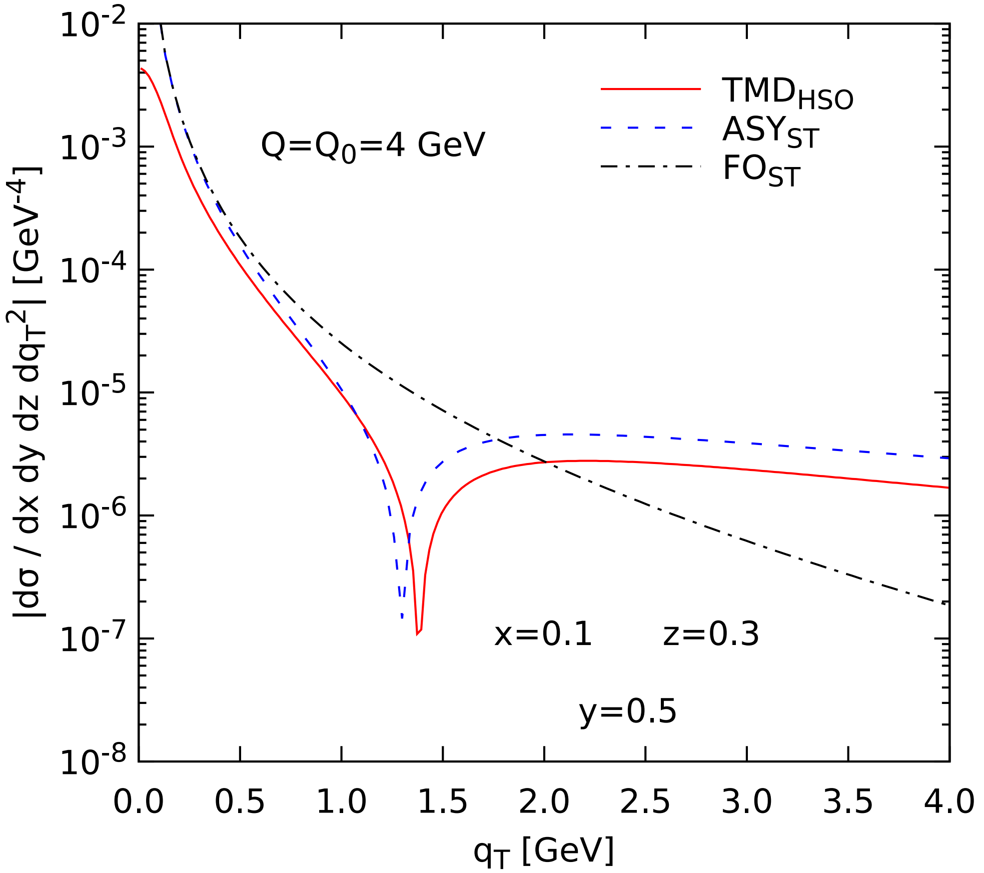

Standard approach
HSO approach (TMD only)
$|d\sigma / dx\, dy\, dz\, dq_T^2|$ [GeV^{-4}]
q_T [GeV]
TMD_{ST}
ASY_{ST}
FO_{ST}
TMD_{HSO}
$Q=Q_0=4$ GeV
$Q=Q_0=20$ GeV
b_{max} = 1.0 GeV^{-1}
M_F = 0.1 GeV
M_D/z = 0.1 GeV
x=0.1 z=0.3
y=0.5

Asymptotic term

The usual asymptotic term

$$\lim_{q_T/Q \to 0} F^{\mathrm{FO}}$$

Still not a good approximation to the TMD term at large q_T

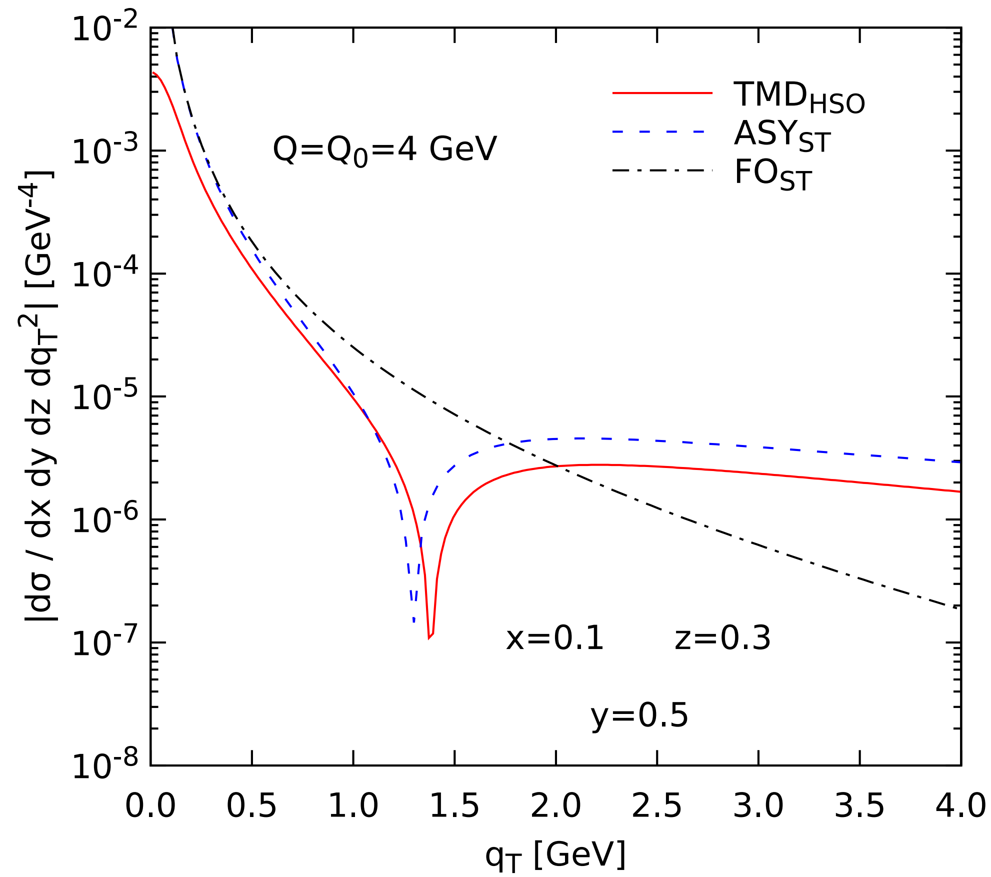


Asymptotic term

The usual asymptotic term

$$\lim_{q_T/Q \to 0} F^{\mathrm{FO}}$$

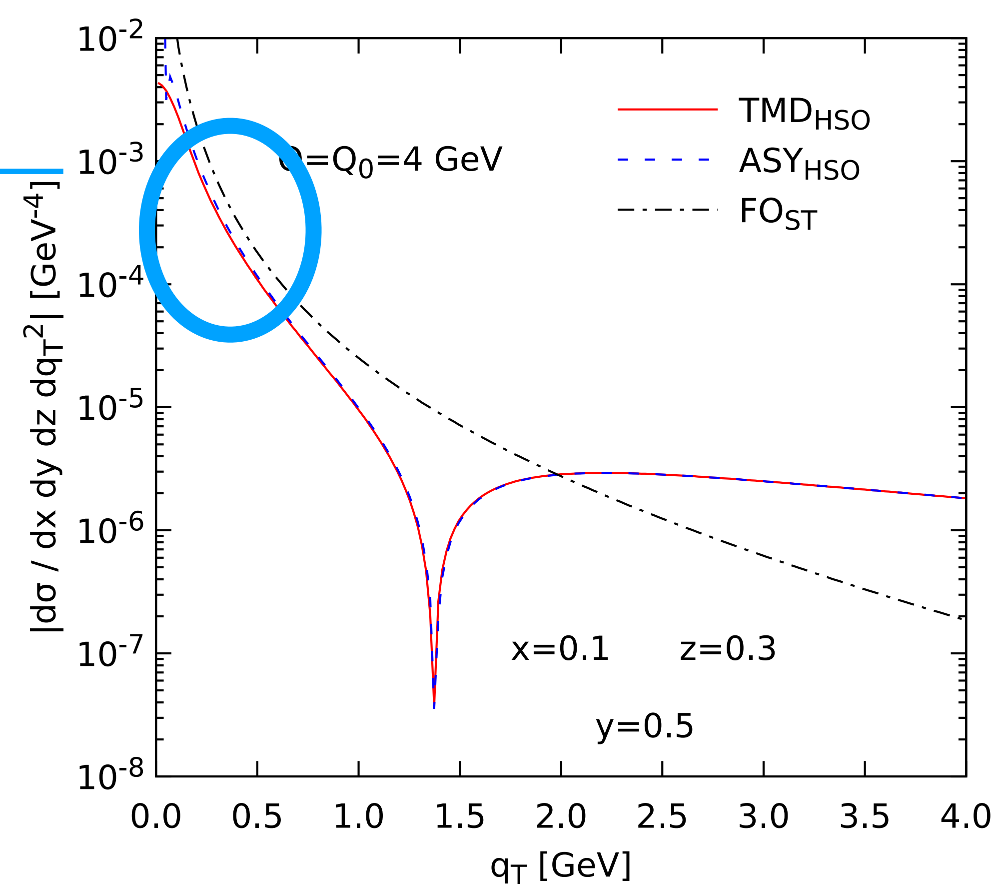
Still not a good approximation to the TMD term at large q_T



We compute instead

$$\lim_{m/q_T \to 0} F^{\mathrm{TMD}}$$

Stays a good approximation to the TMD term at large q_T, from around **this region**



Asymptotic term

The usual asymptotic term

$$\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}}$$

We compute instead

$$\lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_{\mathrm{T}}/Q \to 0} F^{\mathrm{FO}}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_{\mathrm{T}} \to 0} F^{\mathrm{TMD}}\right]^{O(\alpha_s^n)} = O\left(\alpha_s^{n+1} \boxed{, m^2/Q^2)}\right.$$

If using different schemes for collinear functions

Asymptotic term

The usual asymptotic term

$$\lim_{q_{\rm T}/Q \to 0} F^{\rm FO}$$

We compute instead

$$\lim_{m/q_{\rm T} \to 0} F^{\rm TMD}$$

$$\left[\lim_{q_{\rm T}/Q \to 0} F^{\rm FO} \right]^{O(\alpha_s^n)} - \left[\lim_{m/q_{\rm T} \to 0} F^{\rm TMD} \right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2 \right)$$

From two places (fixing the scheme for collinear functions)

Asymptotic term

The usual asymptotic term

$$\lim_{q_\mathrm{T}/Q \to 0} F^{\mathrm{FO}}$$

We compute instead

$$\lim_{m/q_\mathrm{T} \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_\mathrm{T}/Q \to 0} F^{\mathrm{FO}}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_\mathrm{T} \to 0} F^{\mathrm{TMD}}\right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2\right)$$

1) Additional terms in the bracket

$$\begin{aligned}
[f, D] = \; & D^{\mathrm{pert}}(z, z\boldsymbol{q}_\mathrm{T}; \mu_Q; Q^2) f^c(x; \mu_Q) + \frac{1}{z^2} f^{\mathrm{pert}}(x, -\boldsymbol{q}_\mathrm{T}; \mu_Q; Q^2) d^c(z; \mu_Q) \\
& + \int \mathrm{d}^2\boldsymbol{k}_\mathrm{T} \; \Big\{ f^{\mathrm{pert}}(x, \boldsymbol{k}_\mathrm{T} - \boldsymbol{q}_\mathrm{T}/2; \mu_Q; Q^2) D^{\mathrm{pert}}(z, z\left(\boldsymbol{k}_\mathrm{T} + \boldsymbol{q}_\mathrm{T}/2\right); \mu_Q; Q^2) \\
& \qquad - D^{\mathrm{pert}}(z, z\boldsymbol{q}_\mathrm{T}; \mu_Q; Q^2) f^{\mathrm{pert}}(x, \boldsymbol{k}_\mathrm{T} - \boldsymbol{q}_\mathrm{T}/2; \mu_Q; Q^2) \Theta(\mu_Q - |\boldsymbol{k}_\mathrm{T} - \boldsymbol{q}_\mathrm{T}/2|) \\
& \qquad - D^{\mathrm{pert}}(z, z(\boldsymbol{k}_\mathrm{T} + \boldsymbol{q}_\mathrm{T}/2); \mu_Q; Q^2) f^{\mathrm{pert}}(x, -\boldsymbol{q}_\mathrm{T}; \mu_Q; Q^2) \Theta(\mu_Q - |\boldsymbol{k}_\mathrm{T} + \boldsymbol{q}_\mathrm{T}/2|) \Big\} + O\left(\frac{m^2}{q_\mathrm{T}^2}\right)
\end{aligned}$$

Asymptotic term

The usual asymptotic term

$$\lim_{q_\mathrm{T}/Q \to 0} F^{\mathrm{FO}}$$

We compute instead

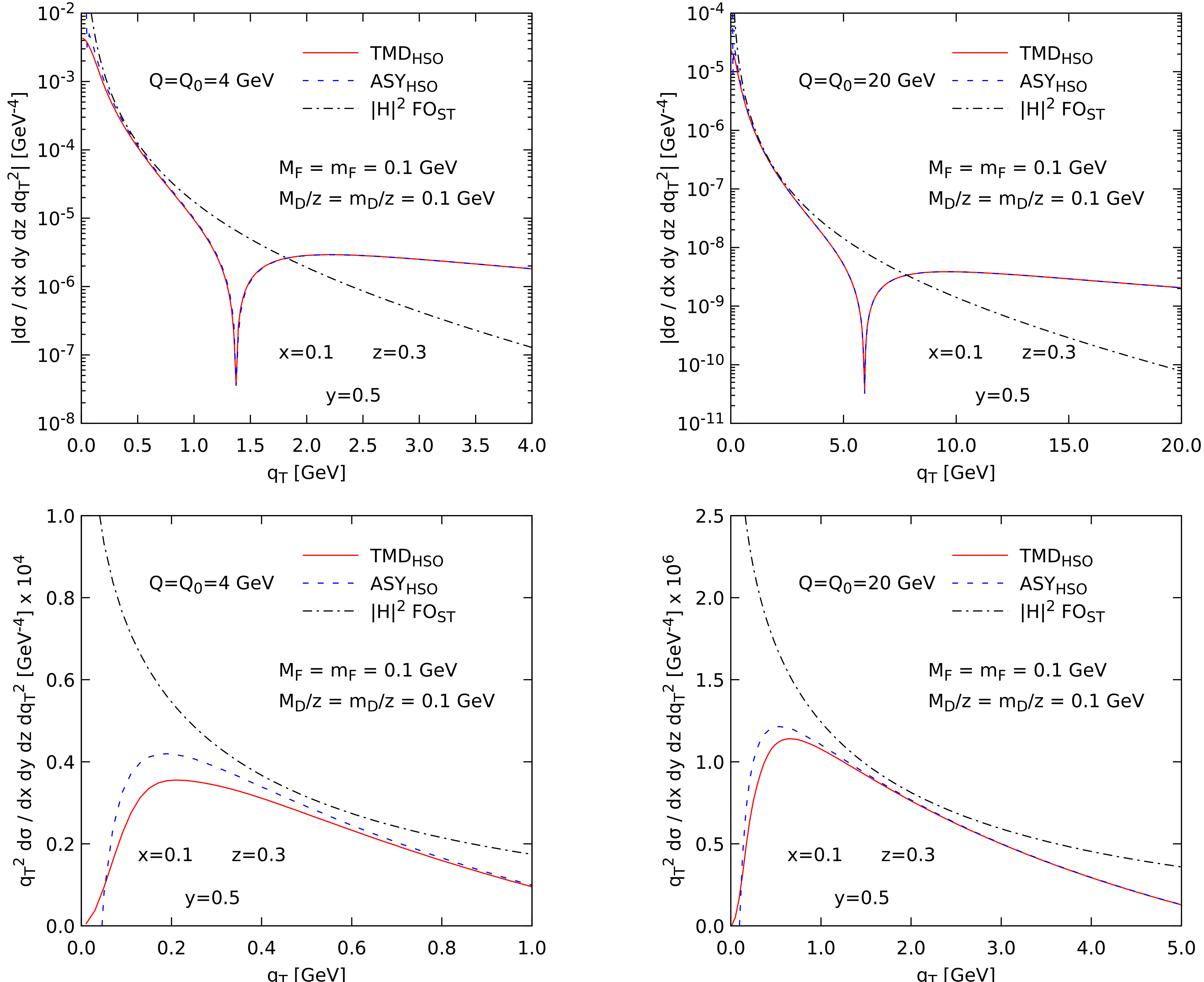
$$\lim_{m/q_\mathrm{T} \to 0} F^{\mathrm{TMD}}$$

$$\left[\lim_{q_\mathrm{T}/Q \to 0} F^{\mathrm{FO}}\right]^{O(\alpha_s^n)} - \left[\lim_{m/q_\mathrm{T} \to 0} F^{\mathrm{TMD}}\right]^{O(\alpha_s^n)} = O\left(\boxed{\alpha_s^{n+1}}, m^2/Q^2\right)$$

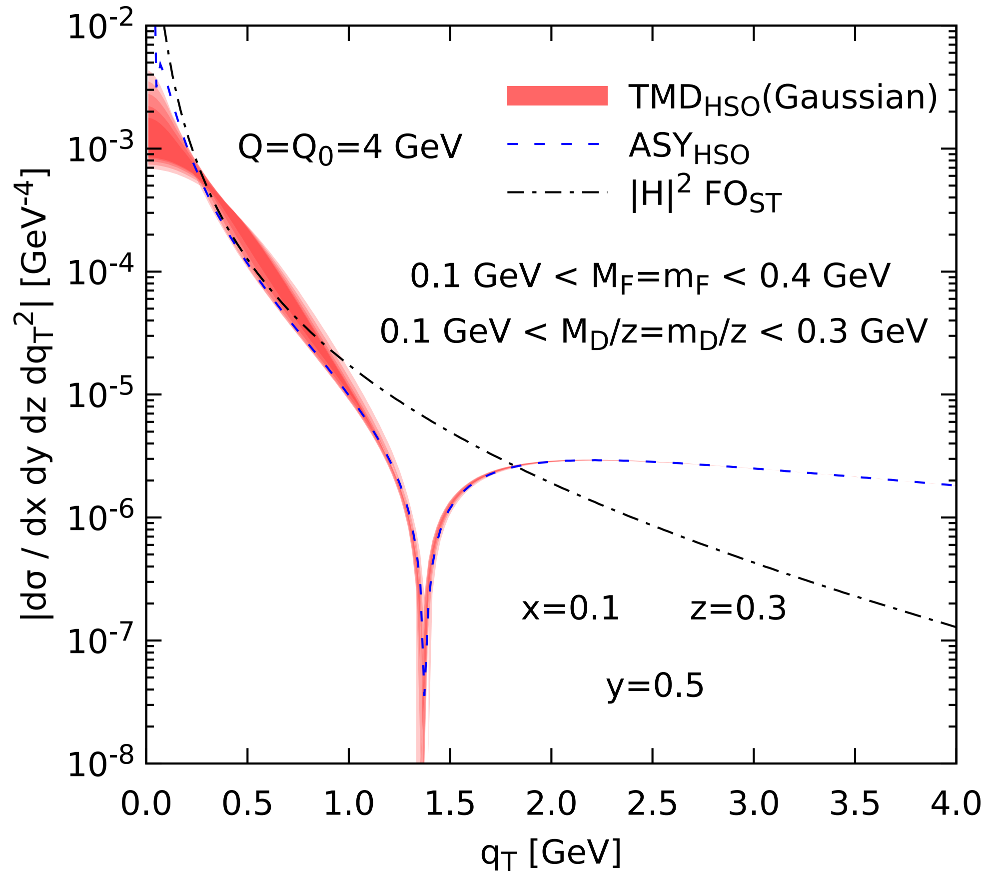
2) Hard coefficient in TMD term

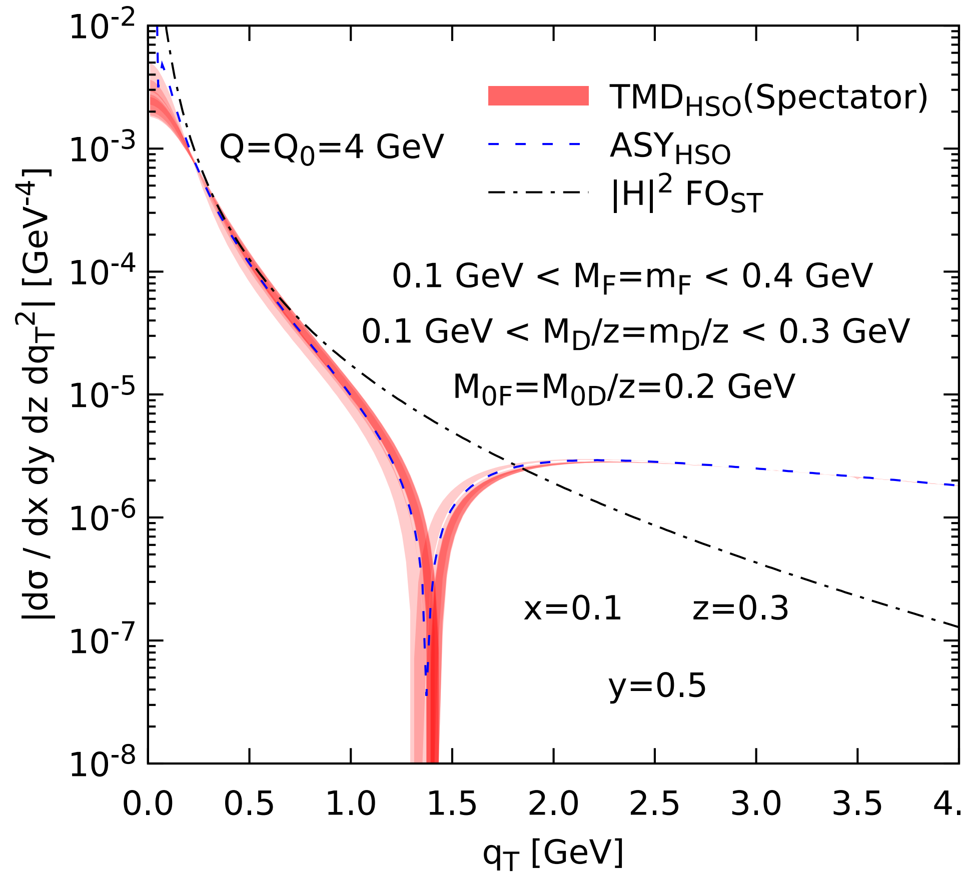
$$F_1^{\mathrm{TMD}} \equiv 2\, z \sum_j \boxed{|H|_j^2} \left[f_{j/p}, D_{h/j}\right] \qquad F_2^{\mathrm{TMD}} \equiv 4\, z\, x \sum_j \boxed{|H|_j^2} \left[f_{j/p}, D_{h/j}\right]$$

HSO approach



HSO approach

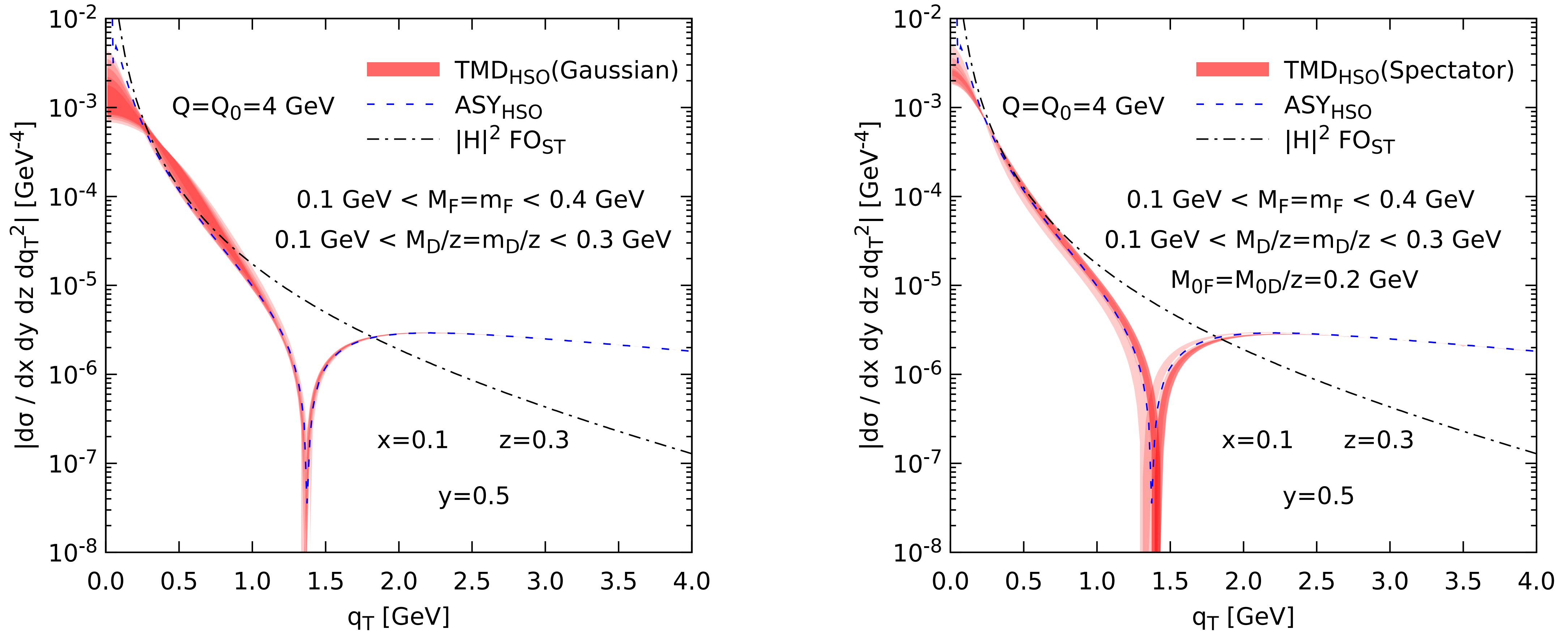




$$f^{\rm Gauss}_{{\rm core},i/p}(x, \boldsymbol{k}_{\rm T}; Q_0^2) = \frac{e^{-k_{\rm T}^2/M_{\rm F}^2}}{\pi M_{\rm F}^2}$$

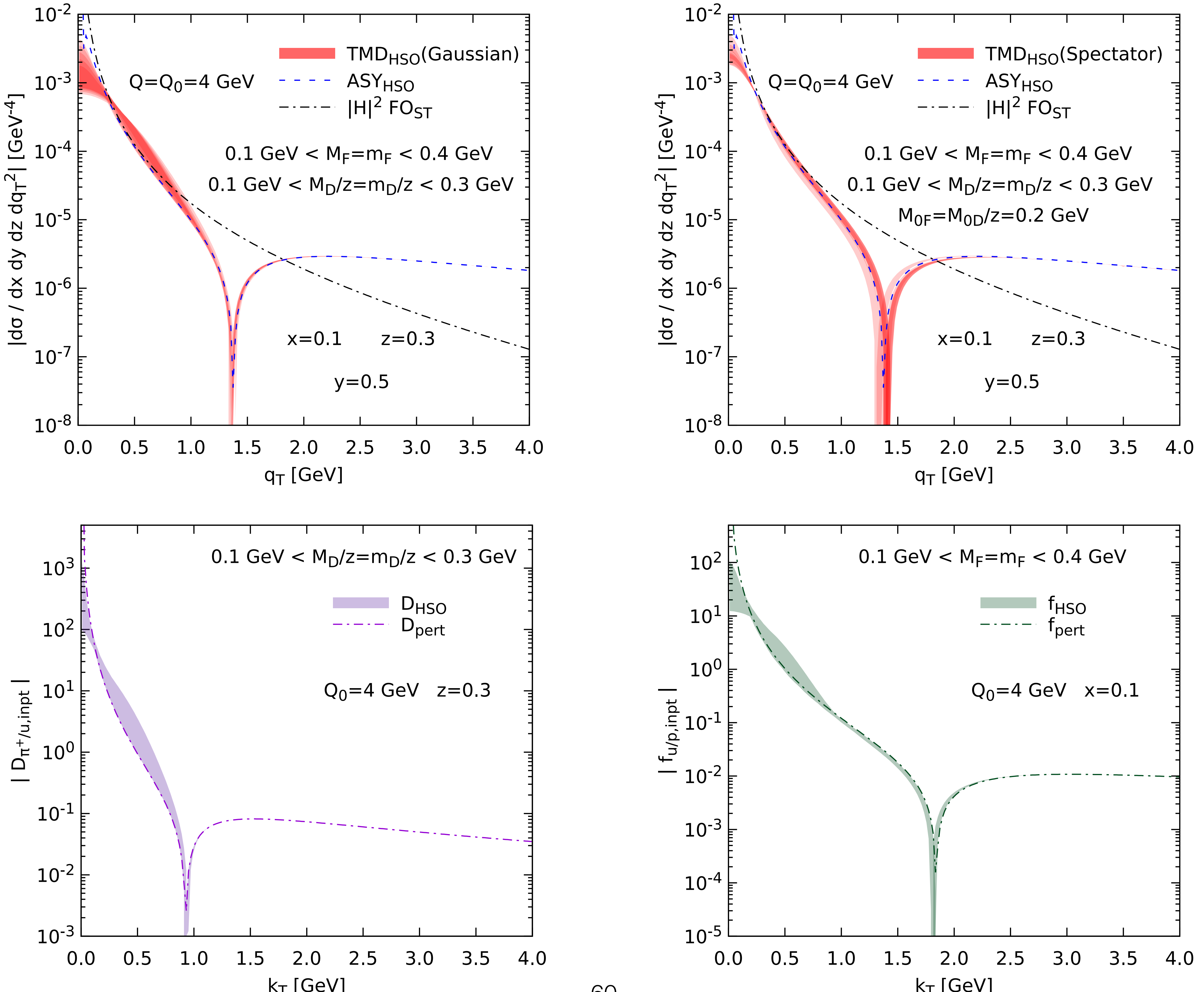
$$f^{\rm Spect}_{{\rm core},i/p}(x, \boldsymbol{k}_{\rm T}; Q_0^2) = \frac{6 M_{\rm 0F}^6}{\pi\left(2M_{\rm F}^2 + M_{\rm 0F}^2\right)} \frac{M_{\rm F}^2 + k_{\rm T}^2}{\left(M_{\rm 0F}^2 + k_{\rm T}^2\right)^4}$$

HSO approach



Consistency of the band with the asymptotic term means the models for TMDs have been made consistent with collinear factorization. In the usual approach, this is the **aim** when embedding the OPE.

HSO approach



✳Standard treatment vs HSO approach.

$\mathbf{b_{max}}$ sensitivity

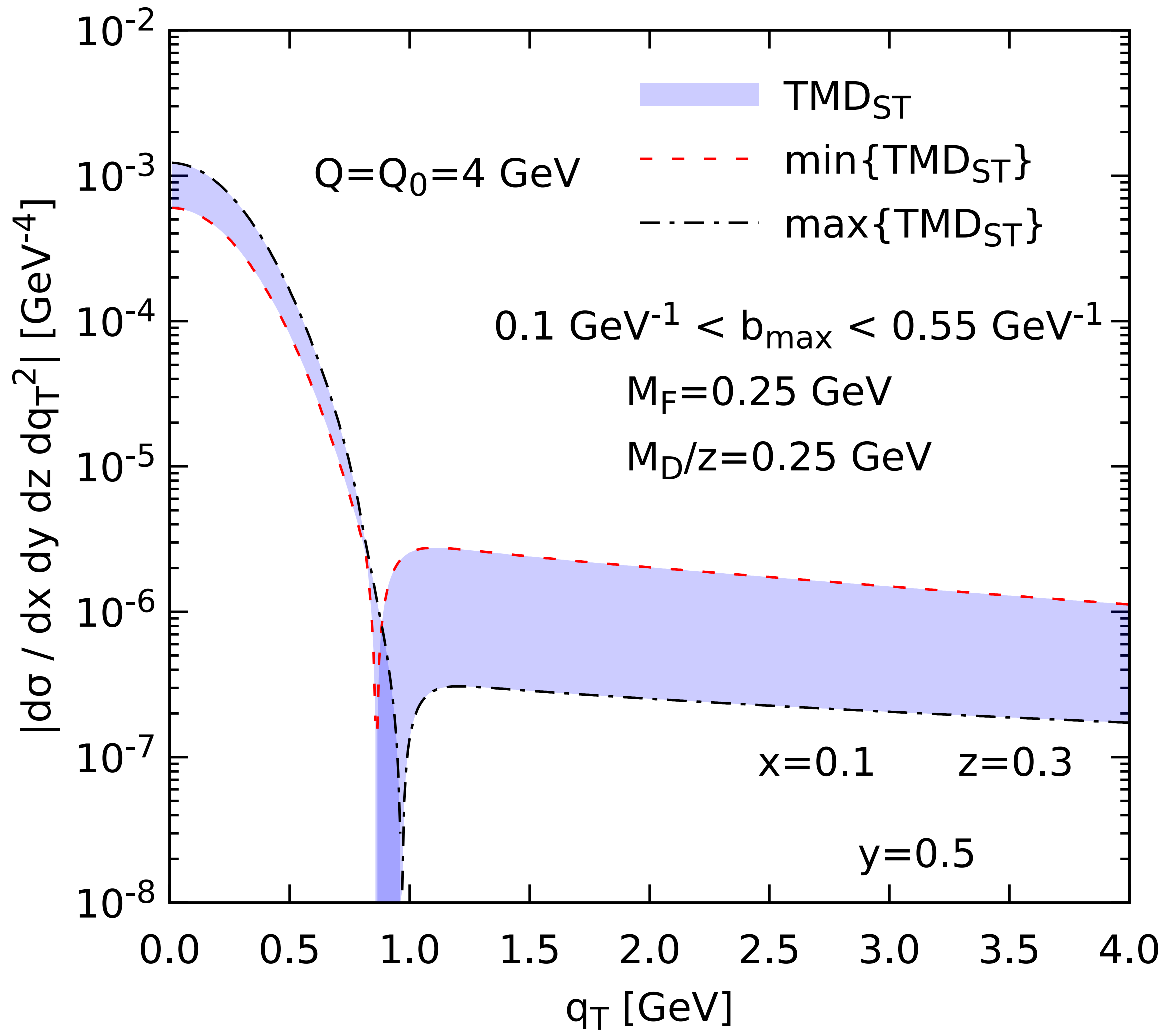
b_* prescription **not used** in HSO. It is instructive though to construct g-functions from HSO approach

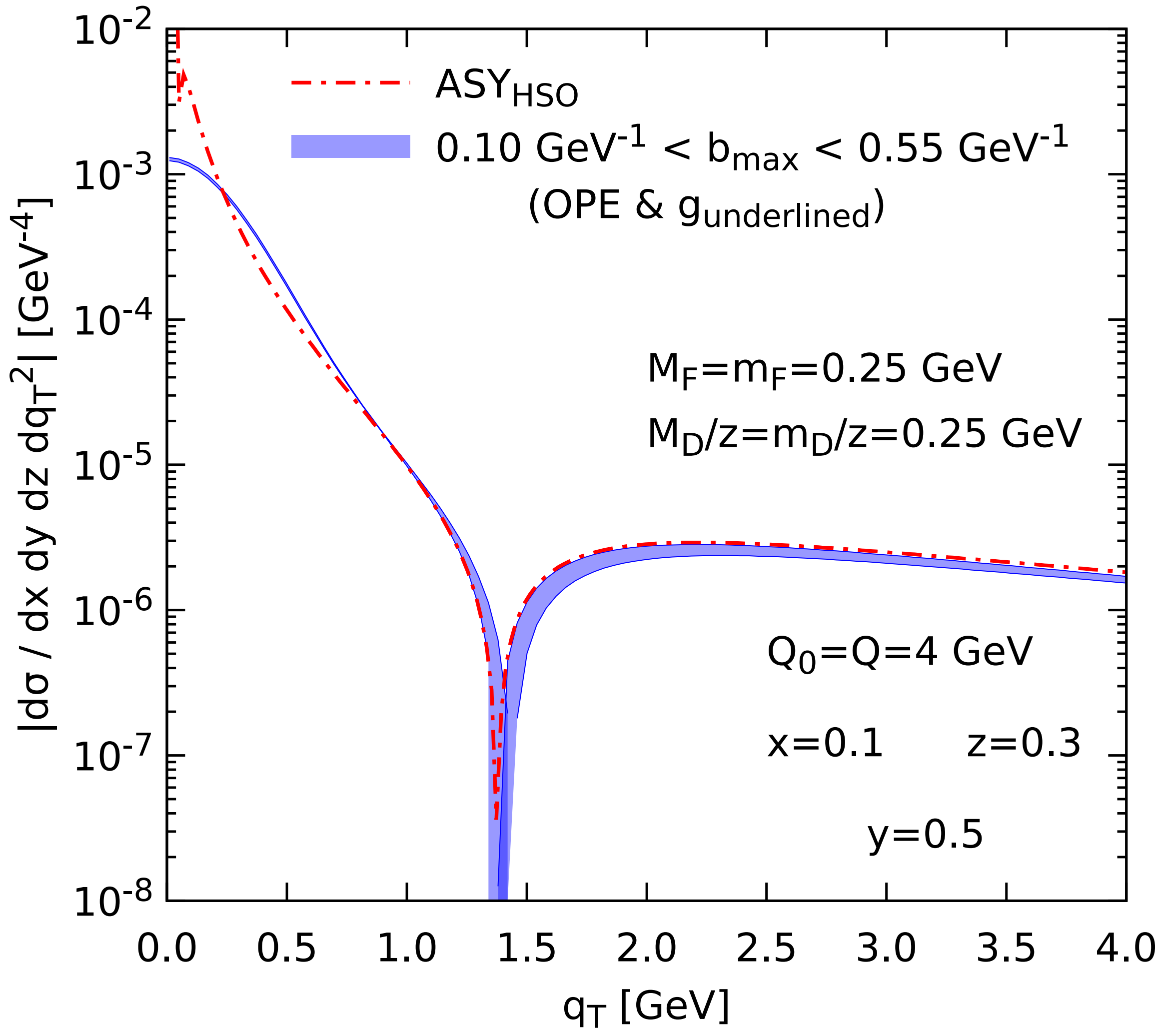
$$-g_{j/p}(x, b_{\mathrm{T}}) \equiv \ln\left(\frac{\tilde{f}_{j/p}(x, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)}{\tilde{f}_{j/p}(x, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)}\right), \qquad -g_{h/j}(z, b_{\mathrm{T}}) \equiv \ln\left(\frac{\tilde{D}_{h/j}(z, \boldsymbol{b}_{\mathrm{T}}; \mu_{Q_0}, Q_0^2)}{\tilde{D}_{h/j}(z, \boldsymbol{b}_*; \mu_{Q_0}, Q_0^2)}\right),$$

$$g_K(b_{\mathrm{T}}) \equiv \tilde{K}(b_*; \mu) - \tilde{K}(b_{\mathrm{T}}; \mu).$$

b_{max} sensitivity

b_* prescription **not used** in HSO. It is instructive though to construct g-functions from HSO approach





Some comparisons