Hadron structure-oriented approach to TMD phenomenology

Fatma Aslan

M. Boglione, J. O. Gonzalez-Hernandez, T. Rainaldi, T. C. Rogers, A. Simonelli

Jefferson Lab

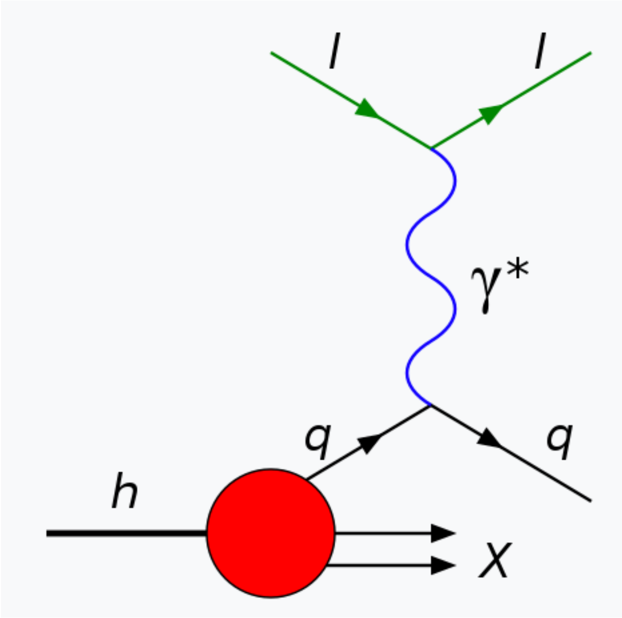
CENTER for NUCLEAR FEMTOGRAPHY

UNIVERSITY OF CONNECTICUT 1881

Outline

- ❑Factorization and Distribution functions
- ❑TMDs and conventional TMD phenomenology
- ❑Problems with the conventional TMD phenomenology
- ❑Hadron Structure Oriented (HSO) TMD phenomenology

DIS Factorization and Collinear Parton Distribution functions, (PDFs), $f(x)$



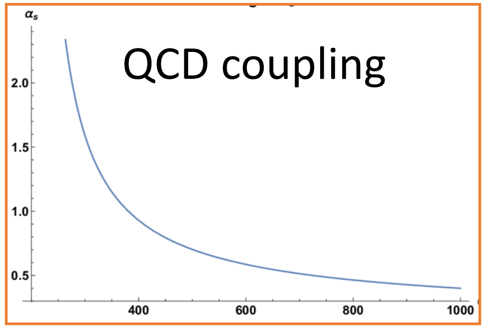
DIS: $l + p \rightarrow l' + X$

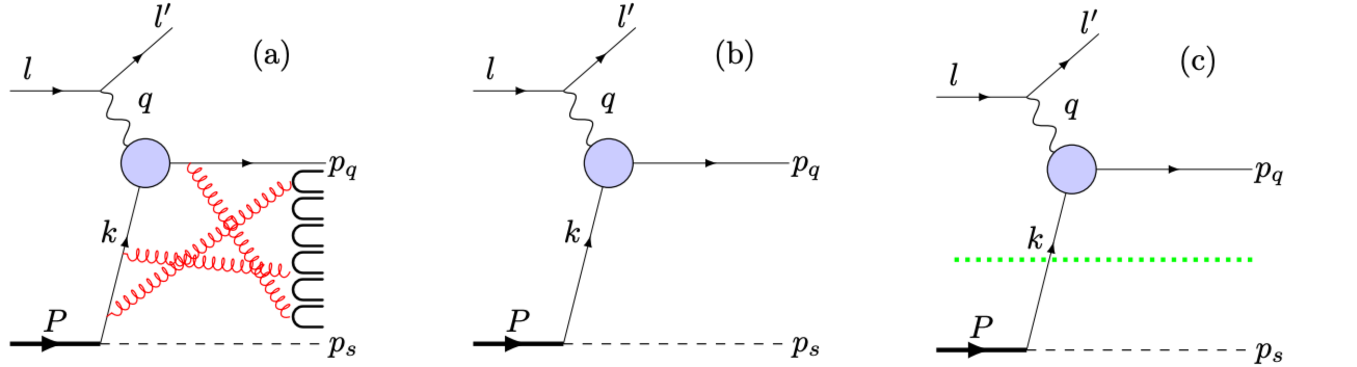
Related to the target (intrinsic properties)
Soft part: The PDF
- Process independent
- Non-perturbative

$$F(x_{bj}, Q) = \int_{x_{bj}}^{1} \frac{d\xi}{\xi} \hat{H}\left(\frac{x_{bj}}{\xi}, \frac{\mu}{Q}\right) f(\xi, \mu) + \mathcal{O}\left(\frac{m}{Q}\right)$$

The physical observable:
Structure Function
- *Expansion over Q*

Related to the parton (collision)
Hard part: Partonic coefficient function
- Process dependent
- Perturbative



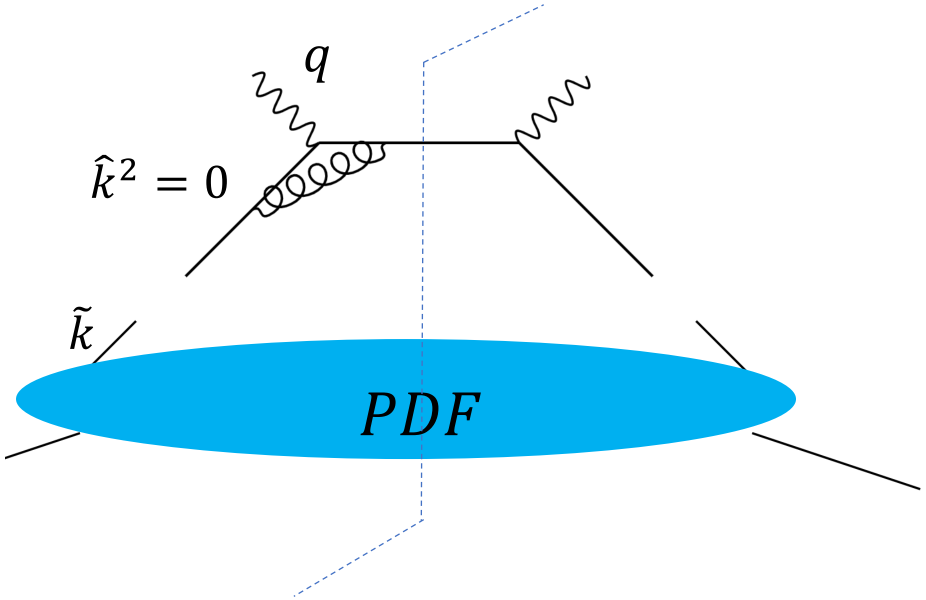


QCD event — leading region — factorization

Feynman's Parton Model

Perturbative expansion
$A + B\alpha(\mu)^2 + \cdots$

$$F(x_{bj}, Q) = \int_{x_{bj}}^{1} \frac{d\xi}{\xi}\, \hat{H}\left(\frac{x_{bj}}{\xi}, \frac{\mu}{Q}\right) f(\xi, \mu) + \mathcal{O}\left(\frac{m}{Q}\right)$$



$Q \to \infty$

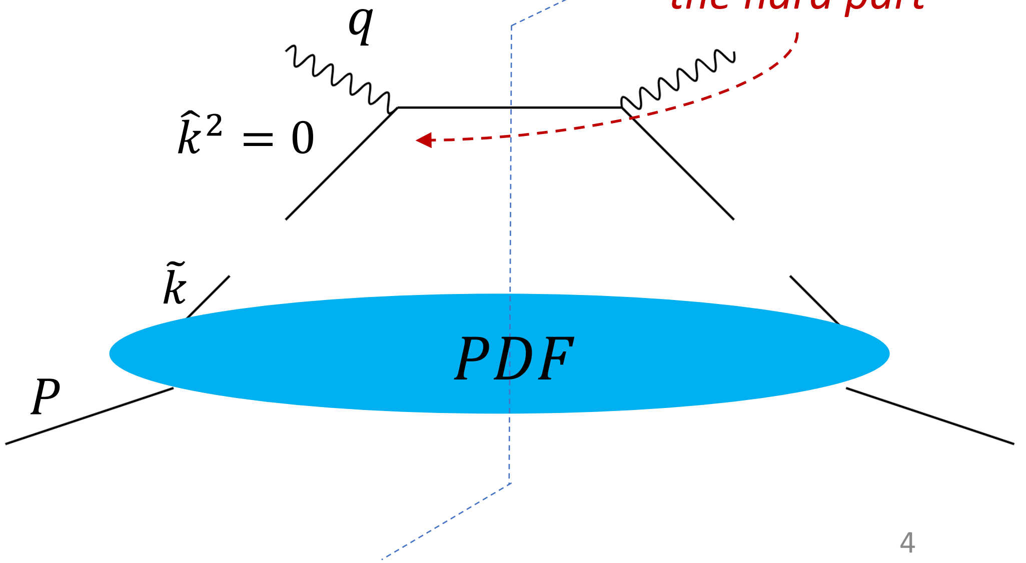
No evolution of the PDF

$$F(x_{bj}) = \int_{x_{bj}}^{1} \frac{d\xi}{\xi}\, \hat{H}\left(\frac{x_{bj}}{\xi}\right) f(\xi)$$

Bjorken scaling
No Q dependence

No QCD interactions
in the hard part

No QCD interactions in the hard part

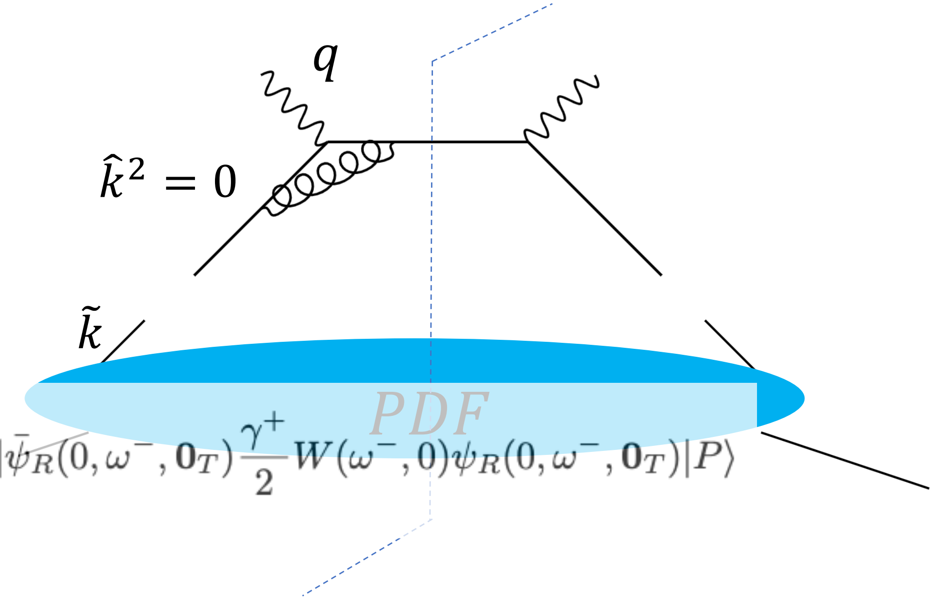


Feynman's Parton Model

Perturbative expansion
$A + B\alpha(\mu)^2 + \cdots$

$$F(x_{bj}, Q) = \int_{x_{bj}}^{1} \frac{d\xi}{\xi}\, \hat{H}\left(\frac{x_{bj}}{\xi}, \frac{\mu}{Q}\right) f(\xi,\mu) + \mathcal{O}\left(\frac{m}{Q}\right)$$

$$f(\xi,\mu) = \int \frac{d\omega^-}{2\pi} e^{-i\xi P^+\omega^-} \langle P|\bar{\psi}_R(0,\omega^-,\mathbf{0}_T)\frac{\gamma^+}{2} W(\omega^-,0)\psi_R(0,\omega^-,\mathbf{0}_T)|P\rangle$$



$Q \to \infty$

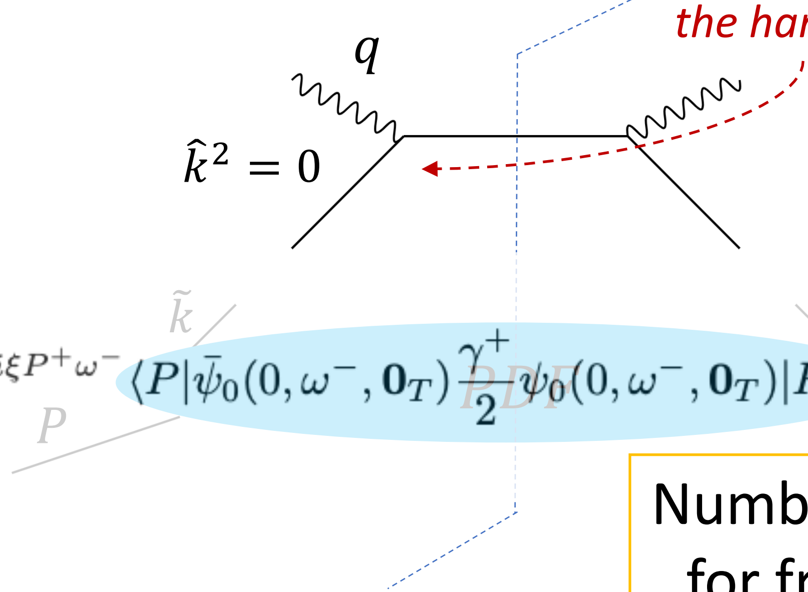
No evolution of the PDF

$$F(x_{bj}) = \int_{x_{bj}}^{1} \frac{d\xi}{\xi}\, \hat{H}\left(\frac{x_{bj}}{\xi}\right) f(\xi)$$

Bjorken scaling
No Q dependence

No QCD interactions in the hard part

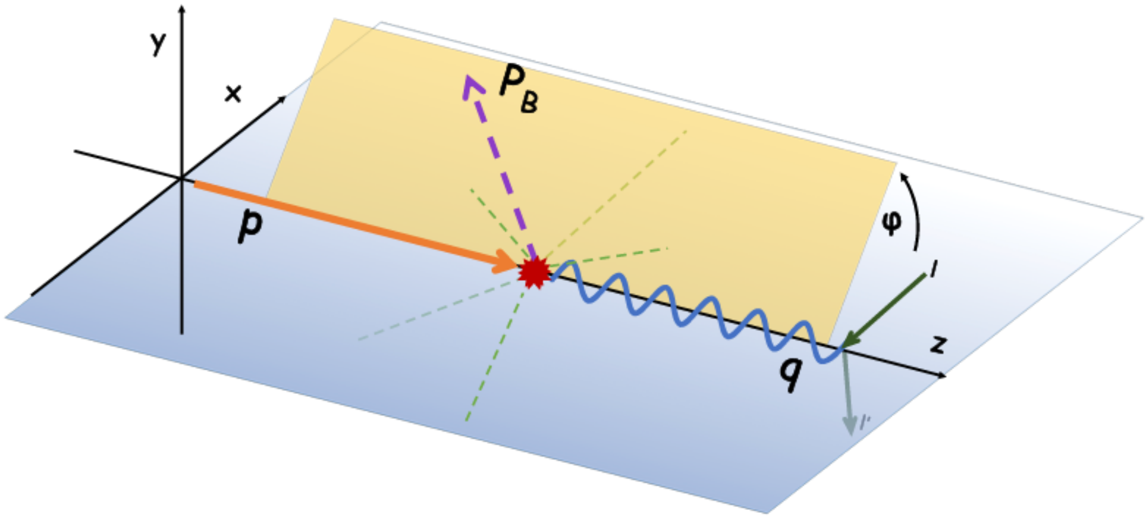
$$f(\xi) = \int \frac{d\omega^-}{2\pi} e^{-i\xi P^+\omega^-} \langle P|\bar{\psi}_0(0,\omega^-,\mathbf{0}_T)\frac{\gamma^+}{2}\psi_0(0,\omega^-,\mathbf{0}_T)|P\rangle \approx \langle P|\, a^\dagger a|P\rangle$$

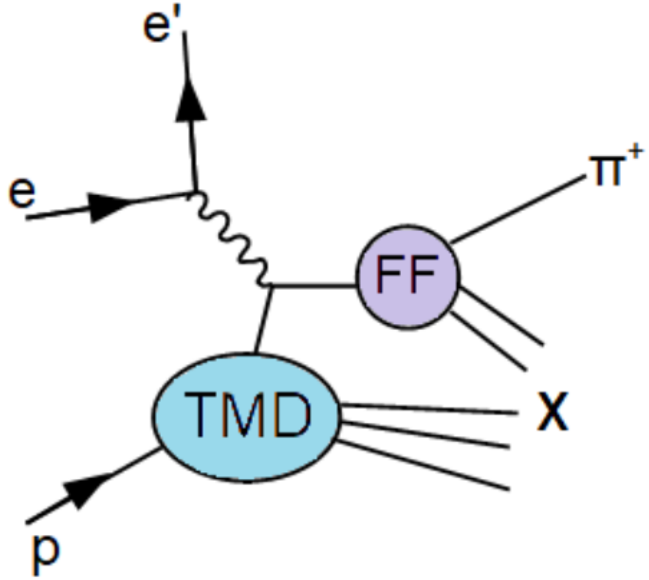


Number density for free fields

SIDIS Factorization and Transverse Momentum Dependent Distributions (TMDs), $f(x, k_T)$, $D(z, k_T)$

SIDIS: $l + p \to l' + P_B + X$



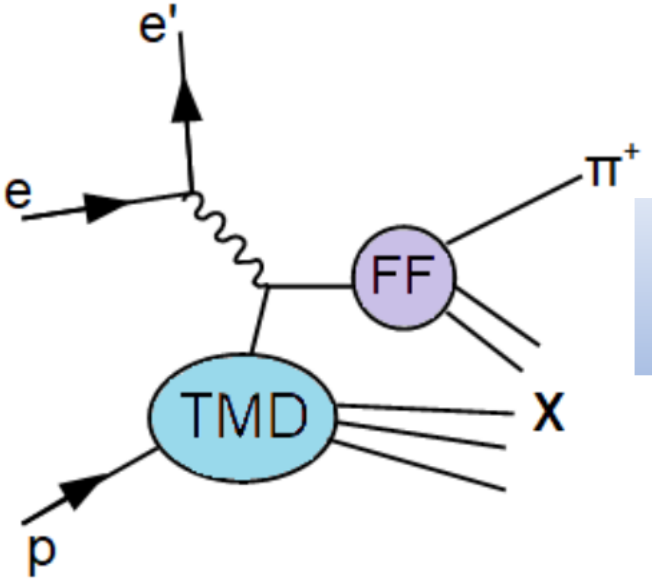


$$W^{\mu\nu}(x, Q, z, \mathbf{P_{BT}}) = \sum_i H_j^{\mu\nu} \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T}\, \underbrace{f_{j/p}(x, \mathbf{k}_{1T}; \mu_Q, Q^2)}_{\text{TMD PDF}}\, \underbrace{D_{h/j}(z, z\mathbf{k}_{2T}; \mu_Q, Q^2)}_{\text{TMD FF}}\, \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

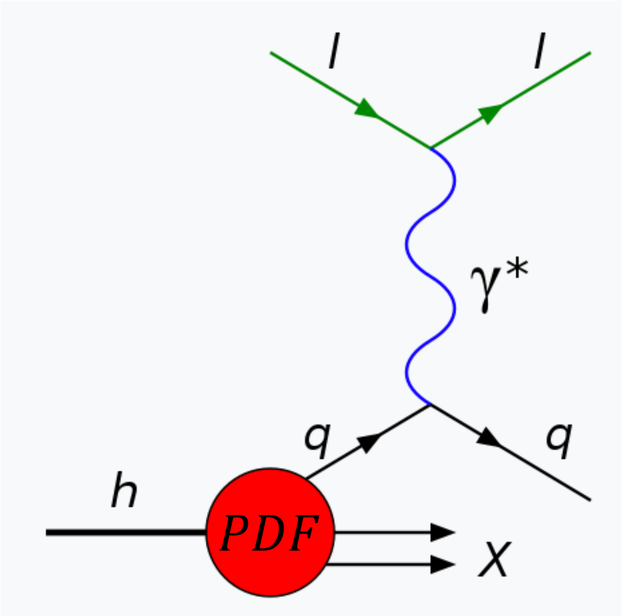
The integral relation

SIDIS: $l + p \to l' + P_B + X$

DIS: $l + p \to l' + X$



Sum all the P_B and integrate over the transverse momenta



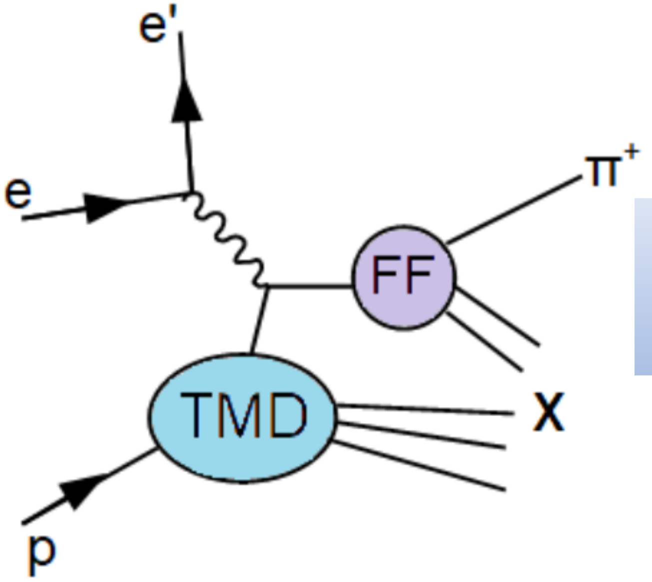
$$A\ probability\ distribution\ is\ like\ P(x) = \int dx\ \ P(x,y)$$

$$\int d^2\boldsymbol{k}_T\ \ f(\xi,\boldsymbol{k}_T) = Divergent\ integral\ ;needs\ to\ be\ renormalized$$

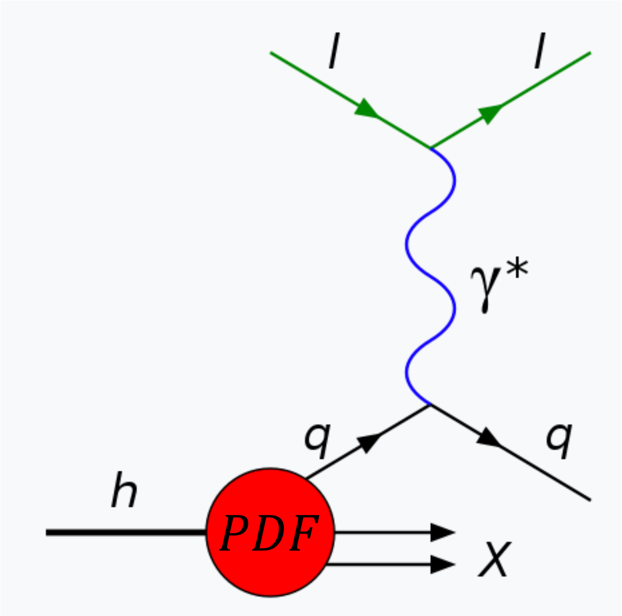
The integral relation

SIDIS: $l + p \rightarrow l' + P_{\text{B}} + X$

DIS: $l + p \rightarrow l' + X$



Sum all the P_B and integrate over the transverse momenta



$$A\ probability\ distribution\ is\ like\ P(x) = \int dx\ \ P(x, y\ )$$

$$\int d^2\boldsymbol{k}_T\ \ f(\xi, \boldsymbol{k}_T) = Divergent\ integral\ ; needs\ to\ be\ renormalized$$

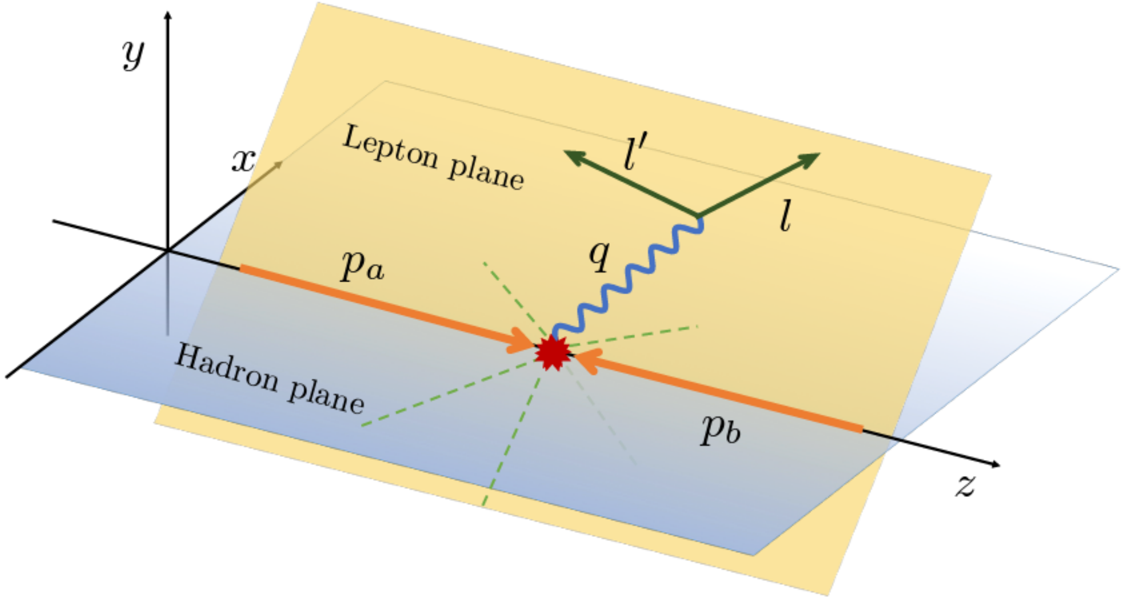
Probability of interacting with a quark that carries longitudinal momentum fraction, ξ

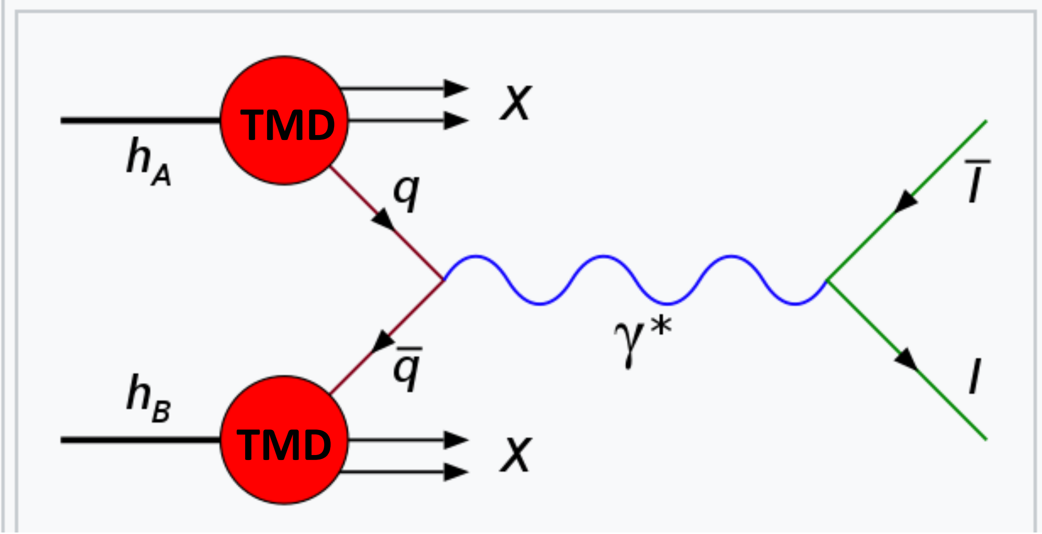
$$f(\xi) = \int d^2\boldsymbol{k}_T\, f(\xi, \boldsymbol{k}_T)$$

Probability of interacting with a quark that carries longitudinal momentum fraction, ξ *and transverse momentum* $\boldsymbol{k}_T$

DY Factorization and Transverse Momentum Dependent Distributions (TMDs), $f(x, k_T)$

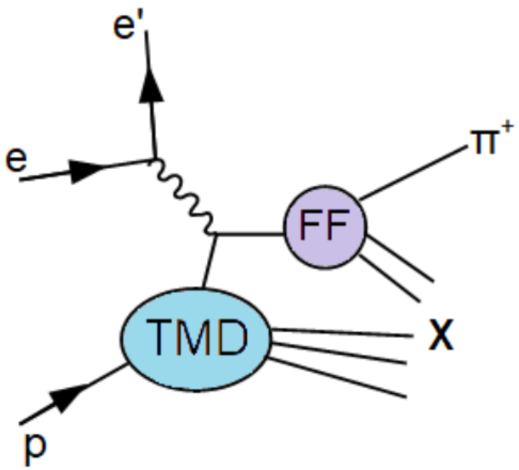
DY: $p_a + p_b \to l + l' + X\,.$





$$W^{\mu\nu}(x_a, x_b, Q, \mathbf{q}_{hT}) = \sum_i H^{\mu\nu}_{j\bar{j}} \int d^2\mathbf{k}_{aT} d^2\mathbf{k}_{bT}\, \underbrace{f_{j/h_a}(x_a, \mathbf{k}_{aT}; \mu_Q, Q^2)}\, \underbrace{f_{\bar{j}/h_b}(x_b, \mathbf{k}_{bT}; \mu_Q, Q^2)}\, \delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{aT} - \mathbf{k}_{bT})$$

Evolution of TMDs



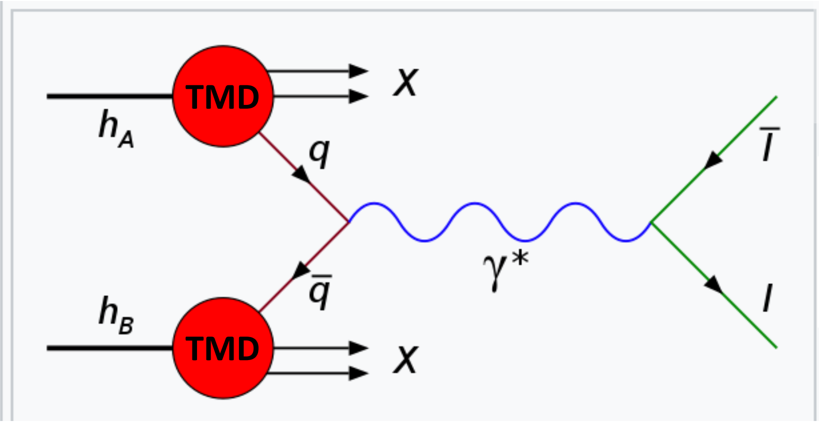
$$W^{\mu\nu}(x,Q,z,\mathbf{P_{BT}}) = \sum_i H_j^{\mu\nu} \int d^2\mathbf{k}_{1T} d^2\mathbf{k}_{2T}\, f_{j/p}(x,\mathbf{k}_{1T};\mu_Q,Q^2)\, D_{h/j}(z,z\mathbf{k}_{2T};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{1T} - \mathbf{k}_{2T})$$

$$\frac{\partial \ln \tilde{f}_{j/p}(x,b_{\rm T};\mu,\zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_{\rm T};\mu)\,,$$

$$\frac{{\rm d}\tilde{K}(b_{\rm T};\mu)}{{\rm d}\ln\mu} = -\gamma_K(\alpha_s(\mu))\,,$$

$$\frac{{\rm d}\ln \tilde{f}_{j/p}(x,b_{\rm T};\mu,\zeta)}{{\rm d}\ln\mu} = \gamma(\alpha_s(\mu);\zeta/\mu^2)$$

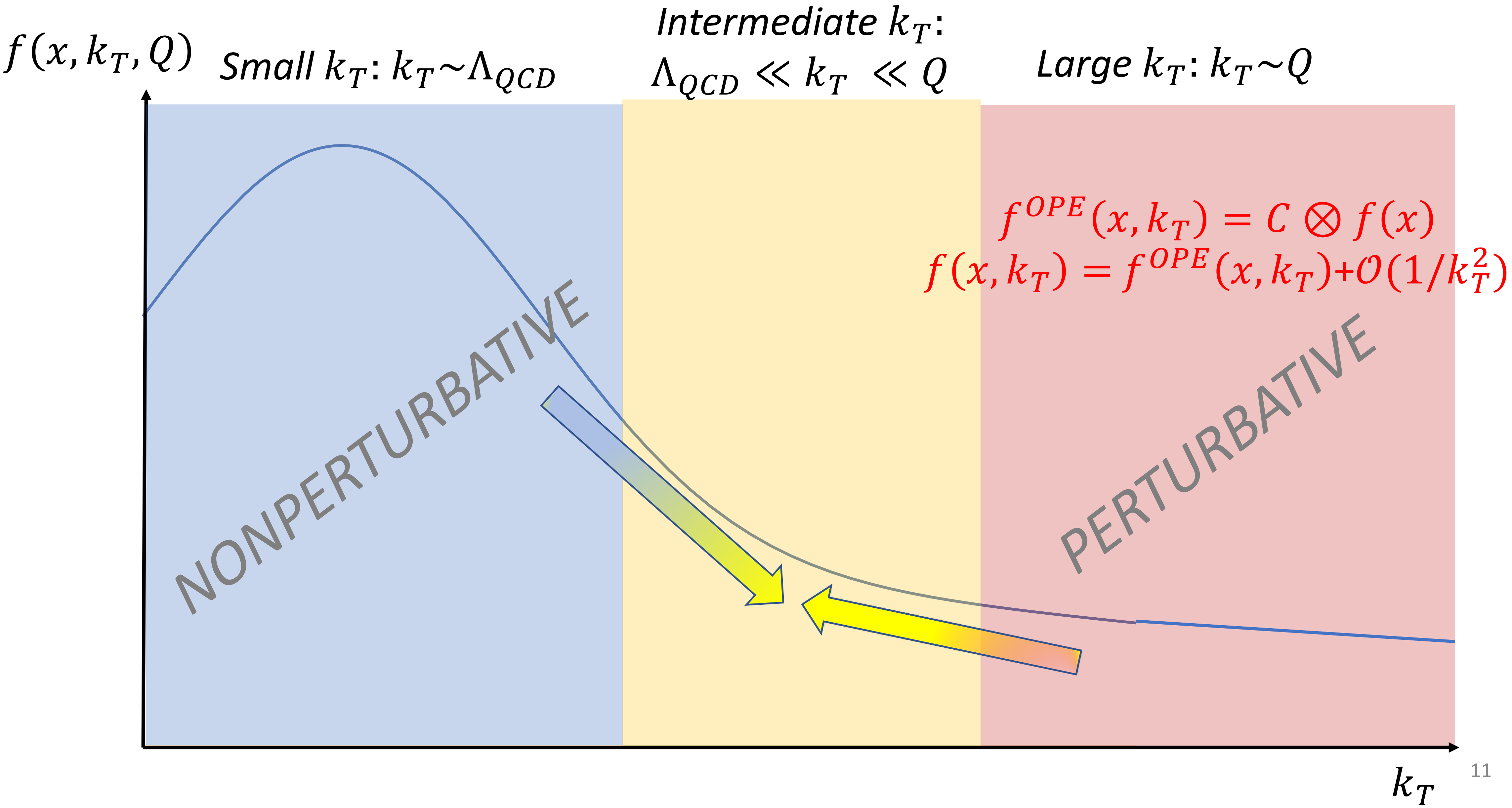
CSS evolution



$$W^{\mu\nu}(x_a,x_b,Q,\mathbf{q}_{hT}) = \sum_i H_{j\bar{j}}^{\mu\nu} \int d^2\mathbf{k}_{aT} d^2\mathbf{k}_{bT}\, f_{j/h_a}(x_a,\mathbf{k}_{aT};\mu_Q,Q^2)\, f_{\bar{j}/h_b}(x_b,\mathbf{k}_{bT};\mu_Q,Q^2)\delta^{(2)}(\mathbf{q}_T + \mathbf{k}_{aT} - \mathbf{k}_{bT})$$

Regions of a TMD

$$f(\xi, \mathbf{k}_T) = \int \frac{d\omega^- d^2\omega_T}{(2\pi)^3} e^{-i\xi P^+\omega^- + i\mathbf{k}_T.\omega_T} \langle P|\bar{\psi}_0(0,\omega^-,\omega_T)\frac{\gamma^+}{2}\psi_0(0,\omega^-,\mathbf{0}_T)|P\rangle$$



Conventional TMD pheno

- Fourier transform $f(x, \boldsymbol{k}_T, Q)$ into the coordinate space

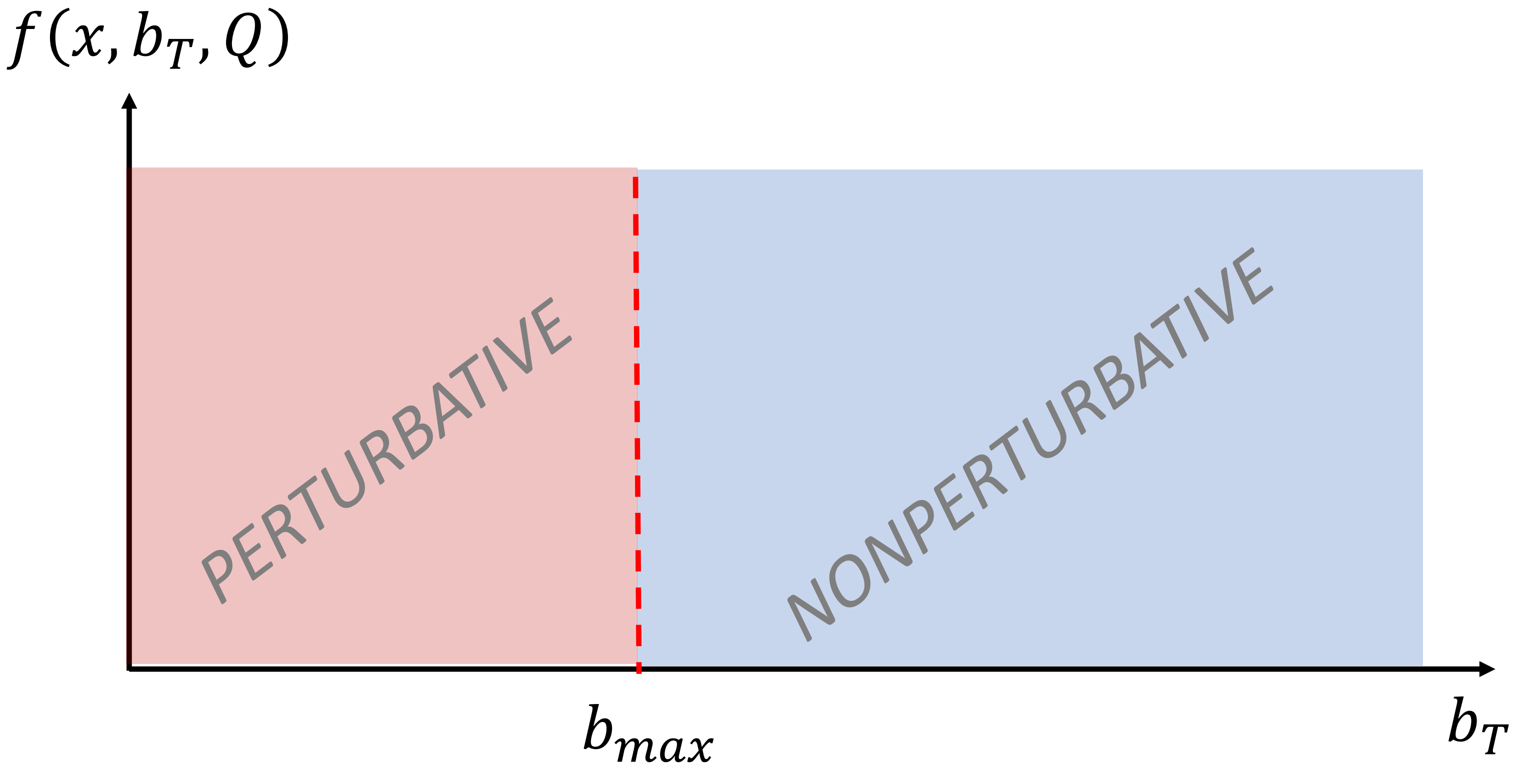
$$\tilde{f}(x, \mathbf{b}_T, Q) = \int d^2\mathbf{k}_T \; e^{-i\mathbf{k}_T.\mathbf{b}_T} f(x, \mathbf{k}_T, Q)$$

Conventional TMD pheno

- Fourier transform $f(x, \boldsymbol{k}_T, Q)$ into the coordinate space
- Choose a demarcation line that separates the nonperturbative and perturbative regions; b_{max}

$$\boldsymbol{b}_*(b_\mathrm{T}) = \begin{cases} \boldsymbol{b}_\mathrm{T} & b_T \ll b_\mathrm{max} \\ \boldsymbol{b}_\mathrm{max} & b_T \gg b_\mathrm{max} \end{cases}$$

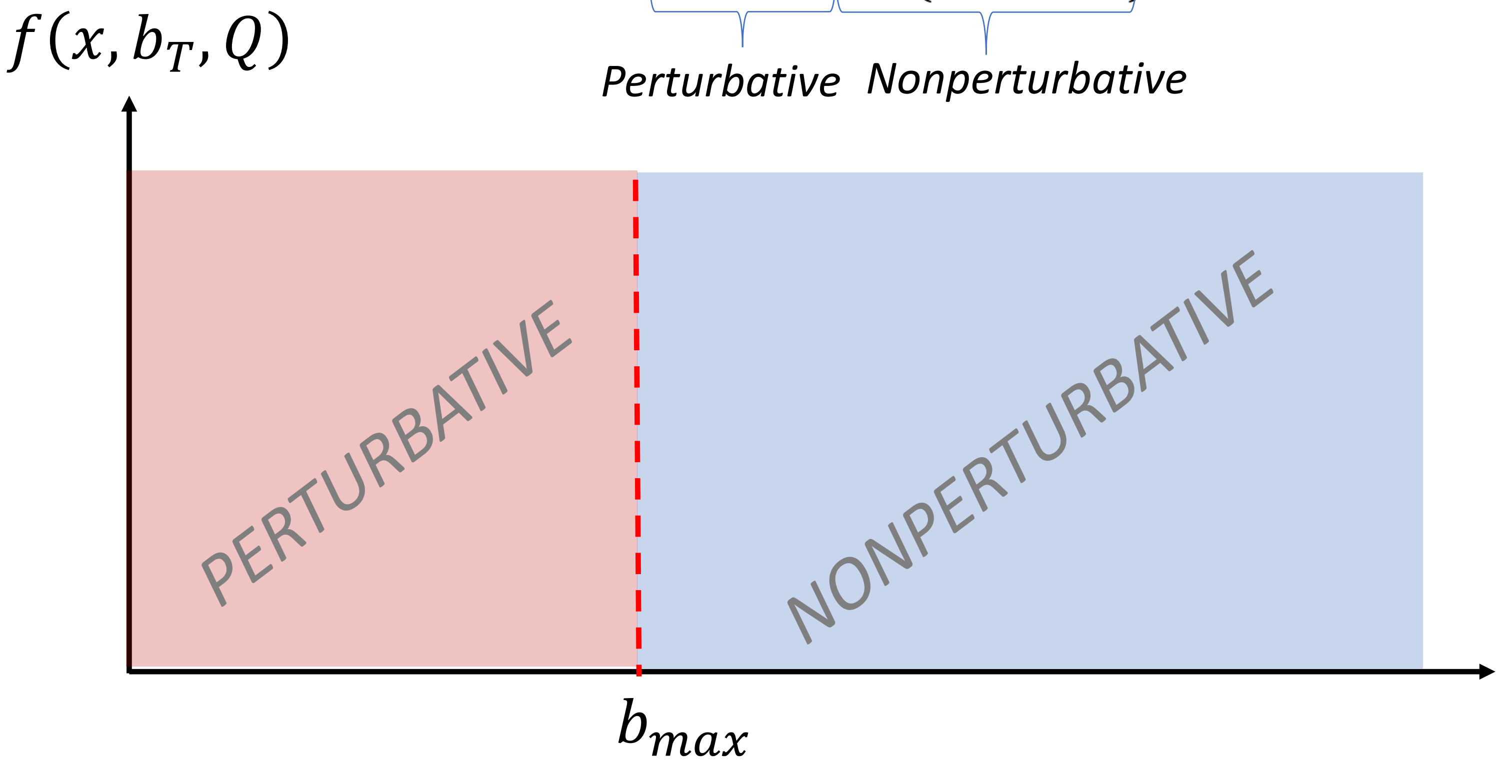
ex: $\boldsymbol{b}_*(b_\mathrm{T}) = \dfrac{\boldsymbol{b}_\mathrm{T}}{\sqrt{1 + b_\mathrm{T}^2 / b_\mathrm{max}^2}}$



Conventional TMD pheno

- Fourier transform $f(x, \boldsymbol{k}_T, Q)$ into the coordinate space
- Choose a demarcation line that separates the nonperturbative and perturbative regions; b_{max}
- Separate $\tilde{f}(x, \boldsymbol{b}_T, Q)$ into perturbative and non-perturbative (NP) parts

$$\tilde{f}(x, \mathbf{b}_T, Q) = \underbrace{\tilde{f}(x, \mathbf{b}_*, Q)}_{\textit{Perturbative}} \underbrace{\exp\{-g(x, \mathbf{b}_T)\}}_{\textit{Nonperturbative}}$$

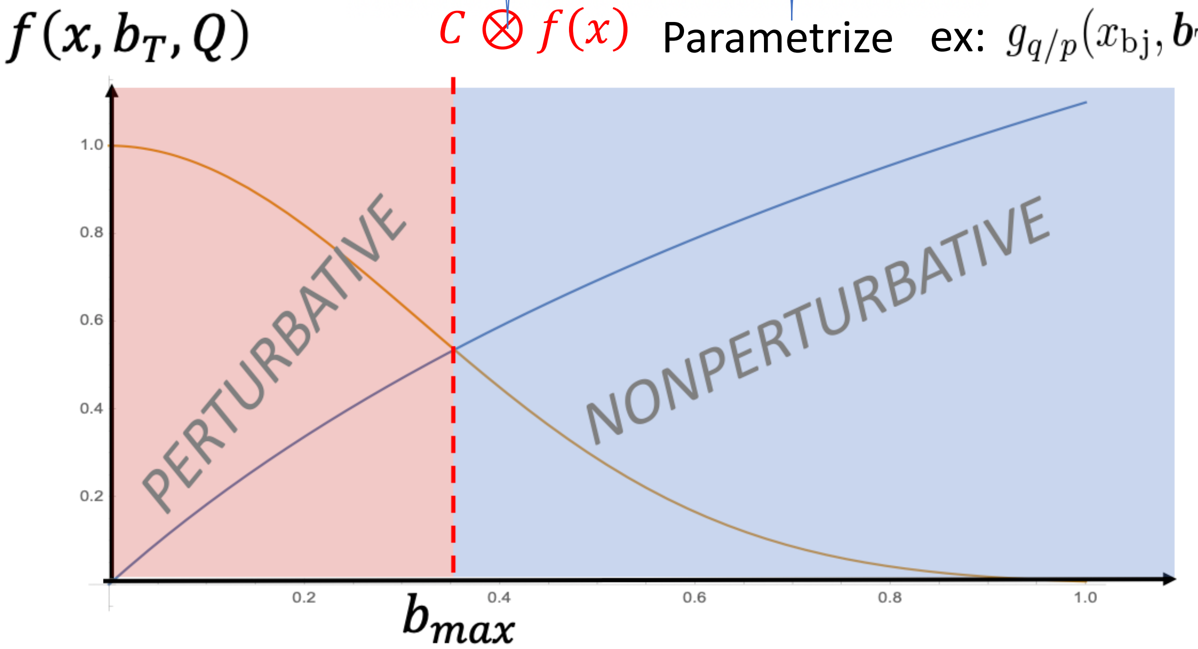


Conventional TMD pheno

- Fourier transform $f(x, \boldsymbol{k}_T, Q)$ into the coordinate space
- Choose a demarcation line that separates the nonperturbative and perturbative regions; b_{max}
- Separate $\tilde{f}(x, \boldsymbol{b}_T, Q)$ into perturbative and non-perturbative (NP) parts
- Calculate the perturbative part using OPE and parameterize the non-perturbative part

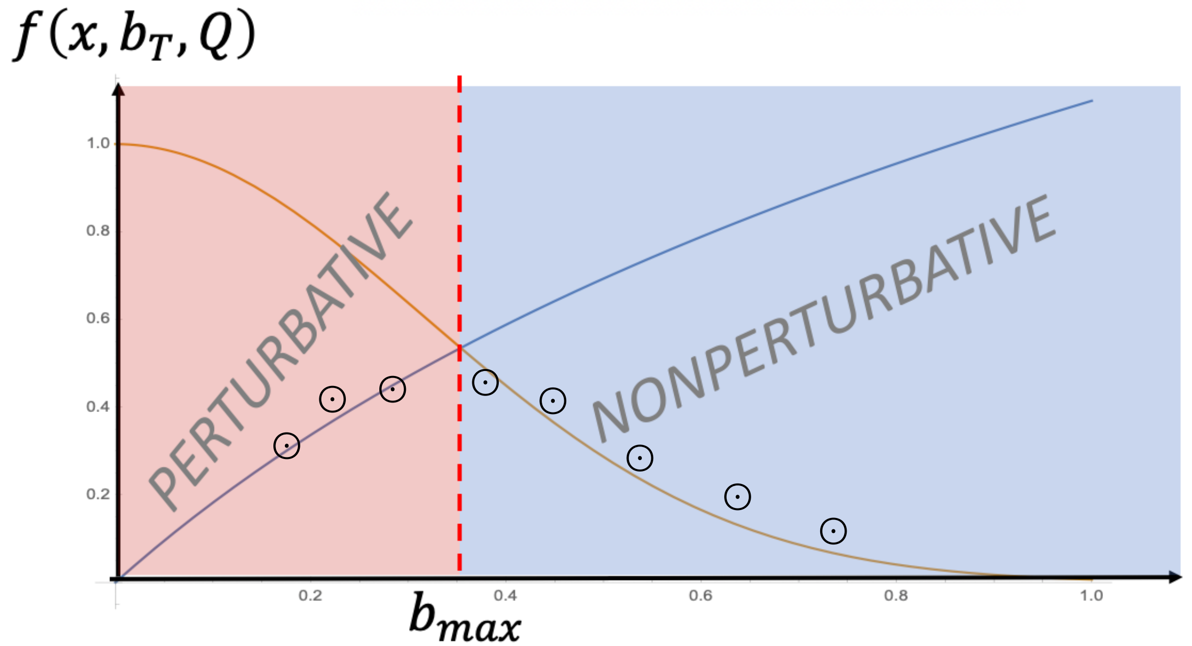
$$\tilde{f}(x, \mathbf{b}_T, Q) = \underbrace{\tilde{f}^{OPE}(x, \mathbf{b}_*, Q)}_{C \otimes f(x)} \underbrace{\exp\left\{ -g(x, \mathbf{b}_T)\right\}}_{\text{Parametrize}} + \mathcal{O}(m^2 b_{max}^2)$$

ex: $g_{q/p}(x_{\rm bj}, \boldsymbol{b}_{\rm T}) \approx g_1 b_{\rm T}$, $g_{q/p}(x_{\rm bj}, \boldsymbol{b}_{\rm T}) \approx g_1 b_{\rm T}^2$.



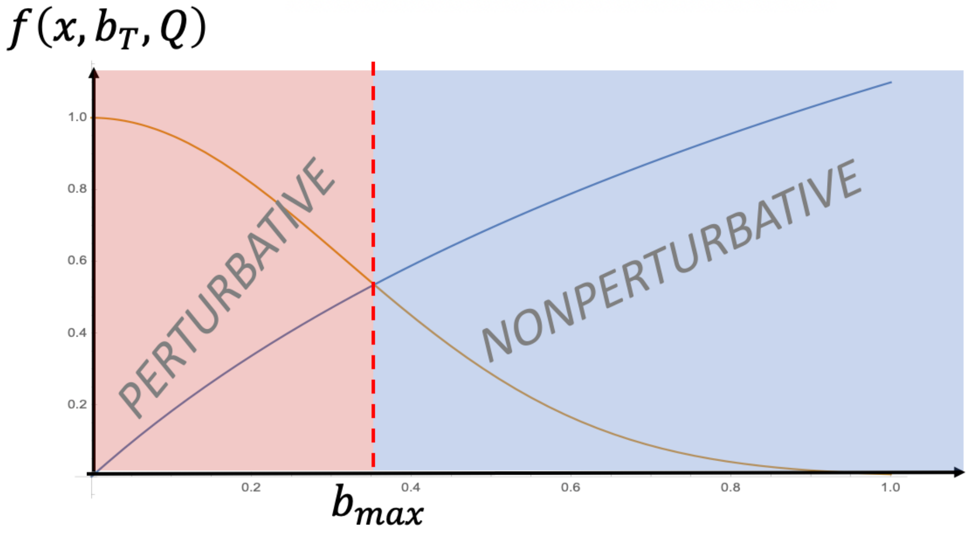
Conventional TMD pheno

- Fourier transform $f(x, \boldsymbol{k}_T, Q)$ into the coordinate space
- Choose a demarcation line that separates the nonperturbative and perturbative regions; b_{max}
- Separate $\tilde{f}(x, \boldsymbol{b}_T, Q)$ into perturbative and non-perturbative (NP) parts
- Calculate the perturbative part using OPE and parameterize the non-perturbative part
- Find the parameters through the fits to the data



Problems of Conventional TMD pheno

- Demarcating the regions as perturbative and nonperturbative in coordinate space and the arbitrary b_{max} dependence



- Matching the TMD parametrizations to the perturbative tail

$$\tilde{f}(x, \mathbf{b}_T, Q) = \tilde{f}^{OPE}(x, \mathbf{b}_*, Q) \exp\{-g(x, \mathbf{b}_T)\} + \mathcal{O}(m^2 b_{max}^2)$$

- Preserving the relation between the TMD and the collinear PDFs to preserve the probabilistic meaning of the TMD

$$f_{i/h}(x) \approx \int \mathrm{d}^2\boldsymbol{k}_\mathrm{T}\, f_{i/h}(x, \boldsymbol{k}_\mathrm{T})\,,$$

- The scale the fits are done and evolution

Hadron structure-oriented approach: The parametrization

A parametrization that

- behaves like $f(x, k_T, Q^2) = \begin{cases} f^{OPE} & \text{at large } k_T \\ \text{NP peak at small } k_T \end{cases}$

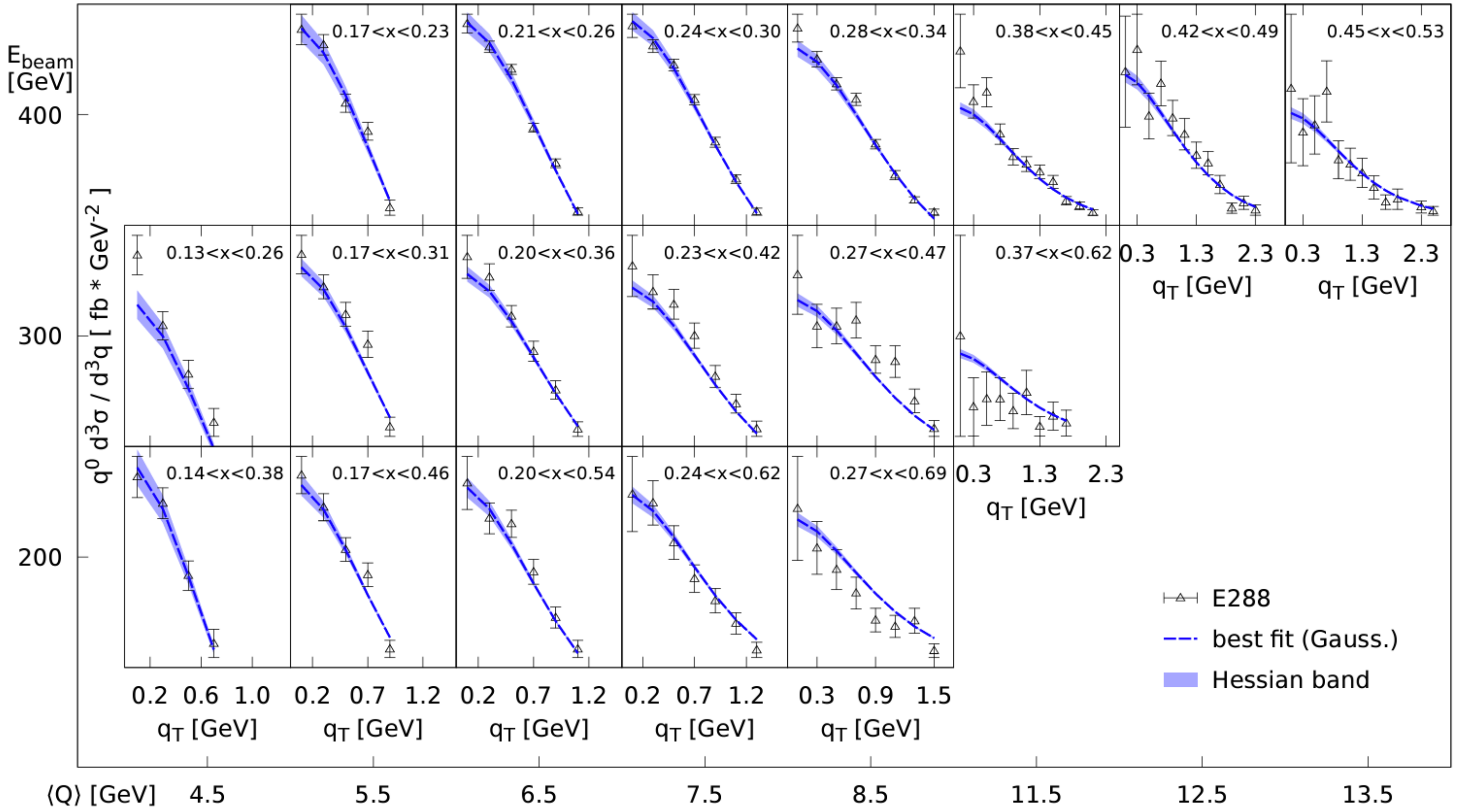
- satisfies $f(x) = \int d^2k_T \, f(x, k_T)$

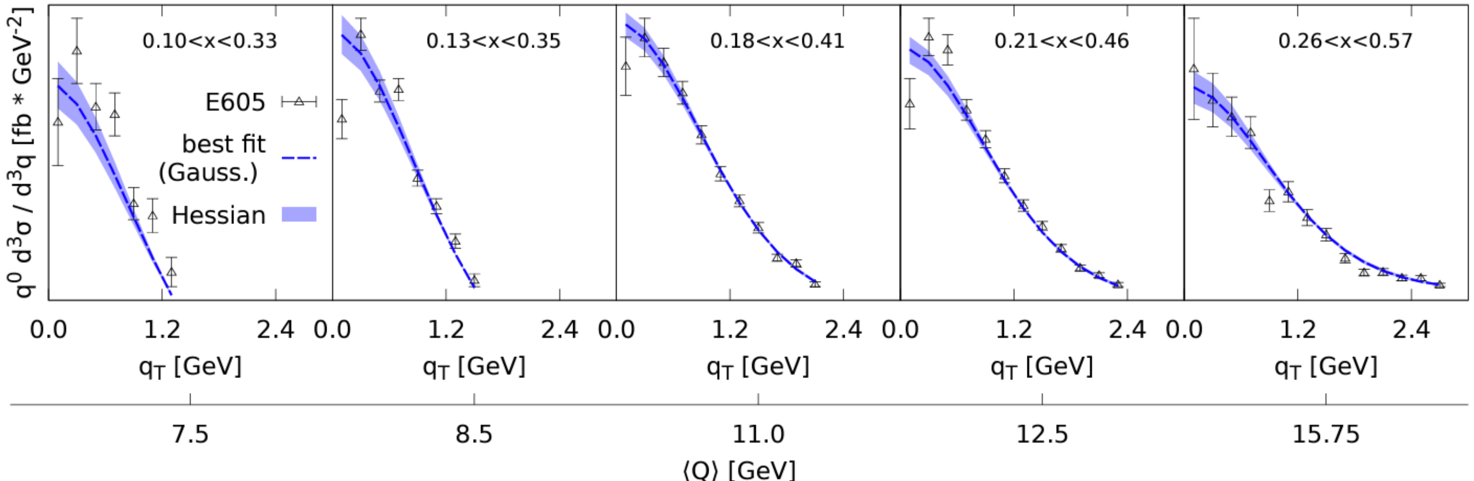
$$f_{\text{inpt},i/p}(x, k_\text{T}; \mu_{Q_0}, Q_0^2) = \frac{1}{2\pi} \frac{1}{k_\text{T}^2 + m_{i,p,A}^2} A_{i/p}(x; \mu_{Q_0}) + \frac{1}{2\pi} \frac{1}{k_\text{T}^2 + m_{i,p,B}^2} B_{i/p}(x; \mu_{Q_0}) \ln\left(\frac{Q_0^2}{k_\text{T}^2 + m_{i,p,L}^2}\right)$$
$$+ \frac{1}{2\pi} \frac{1}{k_\text{T}^2 + m_{g,p}^2} A_{i/p}^g(x; \mu_{Q_0})$$
$$+ C_{i/p} \, f_{\text{core},i/p}(x, k_\text{T}; Q_0^2) \,.$$

$$f_{\text{core},i/p}^{\text{Gauss}}(x, \boldsymbol{k}_\text{T}; Q_0^2) = \frac{e^{-k_\text{T}^2/M_\text{F}^2}}{\pi M_\text{F}^2} \,.$$

$$f_{\text{core},i/p}^{\text{Spect}}(x, \boldsymbol{k}_\text{T}; Q_0^2) = \frac{1}{\pi} \frac{6\,L^6}{L^2 + 2(m_q + x\,M_p)^2} \frac{k_\text{T}^2 + (m_q + x\,M_p)^2}{(k_\text{T}^2 + L^2)^4} \,, \qquad L^2 = (1-x)\Lambda^2 + x M_X^2 - x(1-x) M_p^2 \,,$$

Fitting moderate Q, DY measurements





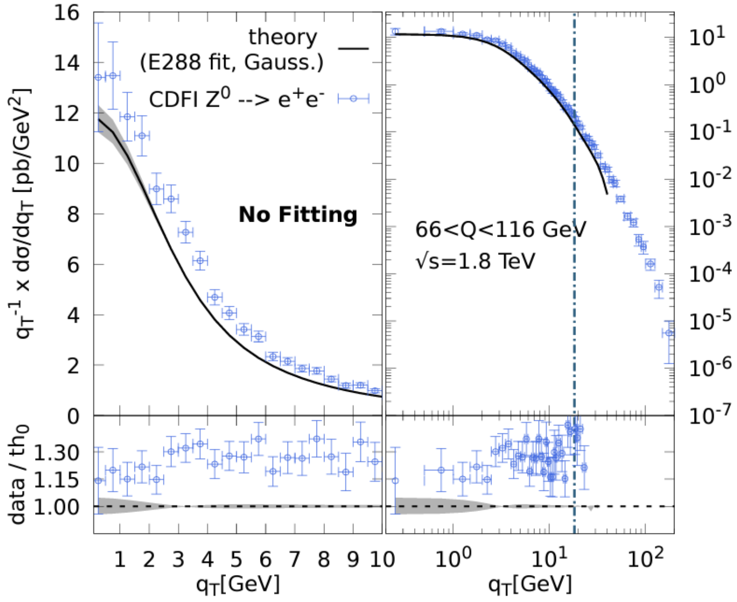
Gaussian fits

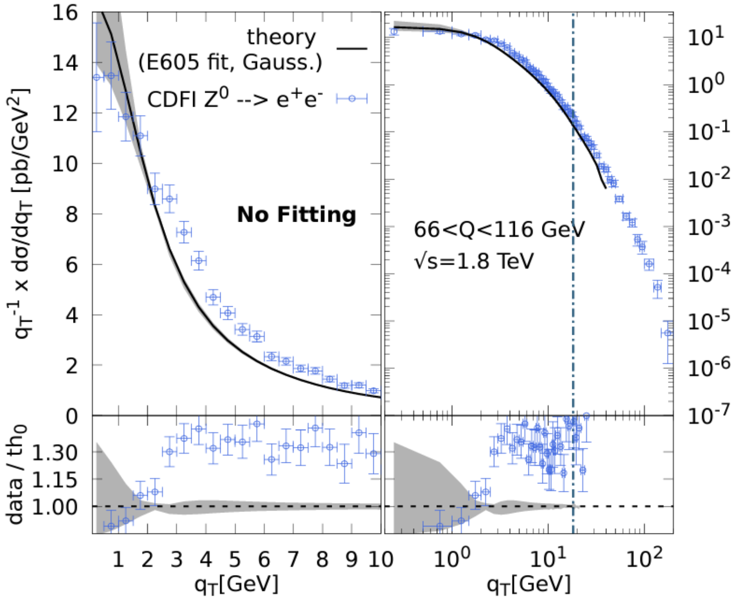
	E288 (130 pts.)	E605 (52 pts.)
$\chi^2_{\rm dof}$	1.04	1.68
M_0 (GeV)	0.0576	0.404
M_1 (GeV)	0.403	0.290
b_K	2.12	0.744
N(nuisance)	1.29	1.28

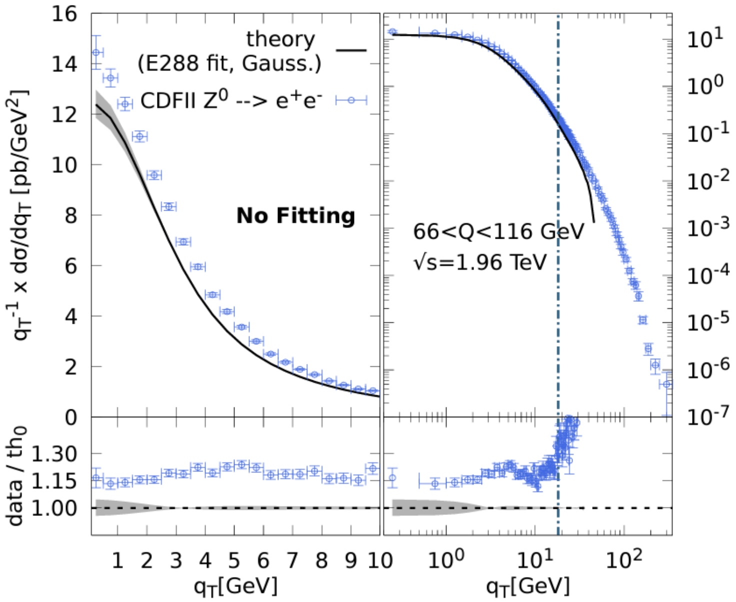
Spectator model fit

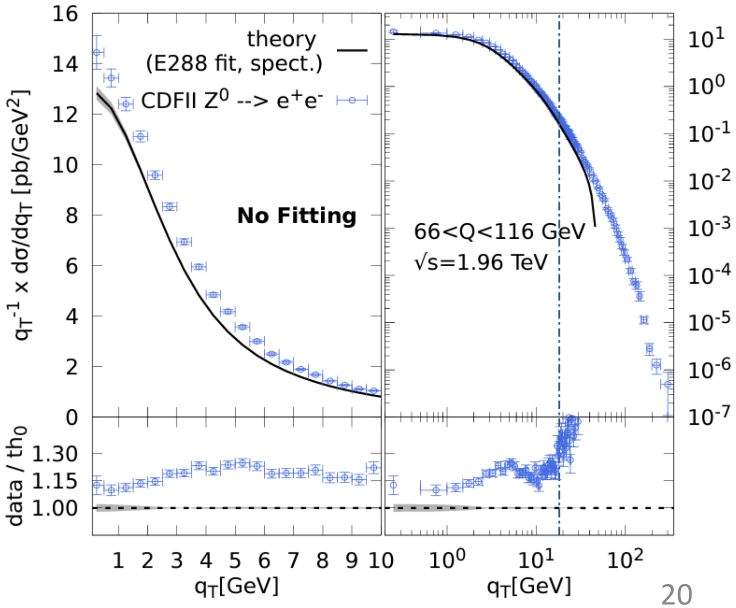
	E288 (130 pts.)
$\chi^2_{\rm dof}$	1.04
Λ (GeV)	0.801
M_X (GeV)	0.438
b_K	1.90
N(nuisance)	1.23

Testing predictions at larger Q









- Conventional TMD pheno
- Hadron structure-oriented approach

- Demarcating the regions as perturbative and nonperturbative in coordinate space and the arbitrary b_{max} dependence
  → There is no abrupt separation of perturbative and nonperturbative regions, no b_{max} dependence

- Matching the TMD parametrizations to the perturbative tail
  → There is no matching issue. The nonperturbative part matches the OPE tail by construction

- Preserving the relation between the TMD and the collinear PDFs to preserve the probabilistic meaning of the TMD
  → The relation between the TMD and the collinear PDFs is preserved and the TMDs have a physical meaning

$$\int \mathrm{d}^2\boldsymbol{k}_\mathrm{T}\, f_{i/h}(x,\boldsymbol{k}_\mathrm{T})\,,$$

$$f_{i/h}(x) \approx \int \mathrm{d}^2\boldsymbol{k}_\mathrm{T}\, f_{i/h}(x,\boldsymbol{k}_\mathrm{T})\,,$$

- Fits are made at high Q and TMDs are evolved from to low Q
  → Fits are made at moderate Q and TMDs are evolved to high Q

Outlook

- Extending the HSO analysis to data covering larger values of Q.

- Broadening the range of processes included at the fitting stage, and to employ a more diverse and sophisticated set of nonperturbative calculations and model assumptions in the treatment of nonperturbative structures.

- Improving the parametrization and get better agreements for the postdictions

- ….

THANK YOU FOR YOUR ATTENTION